THE ROLE OF GRAVITY MODELS IN ESTIMATING THE ECONOMIC IMPACT OF BREXIT

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WP 490
June 2017

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Centre for Business Research, University of Cambridge
Working Paper No. 490

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June 2017

Abstract

The predictions of the Treasury, OECD and IMF for the long-term impact of Brexit remain influential. They provide an important context for the Brexit negotiations and underpin the belief of Scottish and Irish nationalists that Brexit strengthens their case for independence or Irish unity. Because these predictions have received limited scrutiny they are examined in detail in this paper. The bases of the predictions are similar for each of the three organisations. In each case estimates are made of the impact of Brexit on trade and on foreign direct investment. This is followed by an estimate of the knock-on effect on productivity. The OECD and IMF also include an assessment of the impact of lower migration. The aggregate impact of these factors is then fed into a macro-economic model to obtain a forecast for GDP. Much of the final impact depends on the estimate for trade which, in each case, is assessed using a 'gravity model'. Because gravity models are inaccessible to the general public, they are explained here in comprehensible terms. In addition the Treasury's gravity model results are replicated and examined in detail. Our conclusion is that different versions of the model give a range of results and that most versions give a smaller trade impact than that reported by the Treasury, OECD or IMF. In particular, equations which estimate the average impact of EU membership on exports of goods tend to over-predict UK exports to the EU. This implies that the average impact of EU membership applies less to the UK than to the other EU member states. The further implication is that these official predictions of the impact of Brexit are overly pessimistic.

JEL Codes: C54, E24, E44, H24

Keywords: Brexit; Gravity Model; H M Treasury; IMF; Trade: macroeconomic forecasts; OECD

Acknowledgements

We are grateful for insights gained in discussion with Garry Young and Patrick Schneider of the Bank of England. Neither they nor the Bank of England are responsible for the views expressed in this paper. A similar disclaimer must be made for the very helpful comments received from Professor Alan Winters, and Adam Slater both of whom are also not responsible for the views expressed here. We are also grateful to Rachel Wagstaff of the Centre for Business Research for considerable help in the final preparation of this paper.

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Introduction

Almost a year after the EU Referendum and two months after the formal declaration of the UK's intention to withdraw from the EU, much confusion remains on the potential economic impact of Brexit. The UK Treasury, the OECD and IMF all published forecasts of the likely impact of Brexit during the Referendum campaign, as did a range of other forecasting bodies[1]. These included short-term forecasts for the period between the EU referendum in 2016 and the leaving date in 2019, and also longer term forecasts for the decade after the UK has left the EU. Their short-term predictions of the consequences of the decision to withdraw from the EU have thus far been wide of the mark. The negative predictions for the period after 2019 are however still widely believed, although these negative expectations have not been sufficient to deflect either majority UK public opinion or the UK Government from its intention to withdraw from the EU.

Negative expectations for the long-run impact of Brexit are likely to influence the negotiations between the UK and EU authorities, and are already having a clear impact on those in Scotland and Northern Ireland who wish to withdraw from the UK and remain within the EU. Because the likely economic impact on the UK economy remains important, this paper re-examines the evidence presented in the three official reports on the long-term prospects for the UK outside the EU. In each case these three bodies estimated the likely direct impact of Brexit on trade and foreign direct investment (FDI), and the consequent impact on productivity. The OECD and IMF also considered the impact on migration. All three organisations used the same technique, gravity models, to estimate the impact of withdrawal on trade and FDI. Because so much of the evidence for the conclusions on Brexit depends on the gravity model approach, it is on this that we focus in this paper.

The Gravity Model Approach

Leaving the EU will have an economic impact in several ways. Firstly, Brexit is likely to involve higher tariffs, although since both the UK and most EU governments see merit in free-trade, a free-trade agreement between the UK and the remainder of the EU seems likely at some stage. Since the UK already conducts its trade with the EU on a free-trade basis, a new FTA should be easier to agree than the EU's recent agreements with Canada and other countries which required the dismantling of protectionist measures. Secondly, non-tariff barriers in the form of regulatory constraints on trade may grow over time, but they are unlikely to be an important immediate issue since most UK and EU firms currently face the same regulatory framework. Subsequent regulatory changes in both areas may, though, lead to rising non-tariff barriers. Thirdly, since the UK is also likely to leave the customs union there will also be additional costs of administration and potential border delays.

It is possible to assess the likely impact of leaving the EU in a range of different ways. Some attempts have been made to measure the specific impacts such as tariffs on a World Trade Organisation (WTO) basis[2]. Most attempts however use the more aggregate approach of gravity models. This approach attempts to measure the overall benefits to trade or foreign direct investment (FDI) of membership of the EU. It encompasses the whole range of ways in which membership can prove a benefit (or indeed a dis-benefit) without being specific about the benefits of individual measures such as tariffs, regulations, etc. Most approaches use a gravity model to estimate the aggregate benefits of EU membership and then assume that these benefits will be wholly, or largely, reversed, on leaving the EU. Most then add a productivity impact resulting from the changes in trade and FDI, and enter the results into a macro-economic forecasting model (typically NIESR's NiGEM model) to calculate the likely impact on GDP and unemployment.

The Treasury Approach

The Treasury's report on the long-term impact of Brexit starts its analysis by using a gravity model to estimate the increases in trade and FDI which result from membership of the EU and other FTAs. The Treasury's starting point in describing their gravity model for trade in goods is given as:

$$\ln X_{ijt} = \alpha_{ij} + \gamma_t + \alpha_1 \ln\left(Y_{it}Y_{jt}\right) + \alpha_2 \ln\left(POP_{it}\, POP_{jt}\right) + \alpha_3 \ln\left(DIST_{ij}\right) + \alpha_4 COMLANG_{ij} + \alpha_5 COLONY_{ij} + \alpha_5 BORDER_{ij} + \varepsilon_{ijt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where *i* and *j* denote countries, *t* denotes time, and

X_{ijt} denotes trade flows between country *i* and country *j* in year *t*

Y_{it} and Y_{jt} are the GDP of countries *i* and *j* in year *t*

POP_{it} and POP_{jt} are the population of countries *i* and *j* in year *t*

$DIST_{ij}$ is the distance between country *i* and country *j*

$COMLANG_{ij}$ is a dummy variable which equals 1 if the origin and destination countries have a shared language and zero otherwise

$COLONY_{ij}$ is a dummy variable which equals 1 if the origin and destination countries have a shared colonial history and zero otherwise

$BORDER_{ij}$ is a dummy variable which equals 1 if the origin and destination countries share a common border and zero otherwise

γ_t is a set of time dummies for years $1 \ldots T$

α_{ij} is the country-pair fixed effect, one value for each trade-pair

The population variable is often omitted in gravity model work but is presumably included by the Treasury to estimate the impact of per capita GDP over and above the impact of GDP alone. In this form we can expect a negative coefficient indicating that for a given level of GDP there will be less trade when there is more population. In other words while the level of trade may increase with the GDP of exporters and importers, it will be lower if the per capita GDP is lower. A more transparent form of equation (1) would replace POP by GDP/POP. In this case the variable would have a positive coefficient, indicating that trade between two countries will be larger if per capita GDP is higher (while controlling for the level of GDP).

An important feature of this gravity equation is the inclusion of pairwise country and time fixed effects. The country fixed effects, α_{ij}, measure the impact of all influences on each trade-pair flow which are constant over time, such as distance or common language, The time fixed effects, γ_t, measure the impact of common factors across the entire set of countries which vary over time such as prices and trade cycles. The latter is potentially important since it is common to measure the trade flows in current prices while GDP is measured in real purchasing power parity terms.

It is thus obvious that equation (1) cannot be estimated since variables like COMLANG are invariant over time and are thus collinear with the country fixed effects terms α_{ij}. Instead, the Treasury use the following equation:

$$\ln X_{ijt} = \alpha_{ij} + \gamma_t + \alpha_1 \ln\left(Y_{it} Y_{jt}\right) + \alpha_2 \ln\left(POP_{it}\, POP_{jt}\right) + \varepsilon_{ijt} \ldots (2)$$

To estimate how much extra trade is generated through membership of the EU or the European Free Trade Area (EEA), the Treasury add a series of dummy variables to equation (2). These dummies denote whether both countries in a trade-pair are members of the EU at any given date or whether only one of the pair is a member. Further dummies denote membership of the EEA (i.e. Norway and Iceland), or membership of an FTA with the EU. This gives the equation the Treasury actually estimate:

$$\ln X_{ijt} = \alpha_{ij} + \gamma_t + \alpha_1 \ln\left(Y_{it}Y_{jt}\right) + \alpha_2 \ln\left(POP_{it}\, POP_{jt}\right) + \beta_1 EU2_{ijt} + \beta_2 EU1_{ijt} + \beta_3 EEA_{ijt} + \beta_4 FTA_{ijt} + \varepsilon_{ijt}$$

………………… (3)

where:

- EU1 is a dummy variable which equals 1 if only one country is a member of the euro area at time t and zero otherwise
- EU2 is a dummy variable which equals 1 if both the origin and destination countries are members of the euro area at time t and zero otherwise
- EEA is a dummy variable which equals 1 if the origin country is a member of the European Free Trade Area.
- FTA is a dummy variable which equals 1 if the origin country is a member of a FTA with the EU

The dummy variables for membership of the EU or other FTAs can be estimated (i.e. are not collinear with the country fixed effect dummies) as long as the data includes periods in which countries were not members of the FTA. An analysis with data from 1948 to 2013 includes a number of years in which all of the countries in the dataset were not members of the EU or the other FTAs. Shorter periods, which we examine below, would provide only estimates of the impact of FTA membership for those countries which joined the FTA during the period of analysis. For instance, an analysis with data from 2000-2013 would estimate the impact of EU membership only for the East European and other countries which joined the EU after 2000[3].

The Treasury report gives limited information about the nature of their analysis[4]. The Report states that IMF data on trade flows is from the Glick and Rose database covering 200 countries over the period 1948-2013. With 39,800

(=200 x 199) country pairs over 65 years this would give 2.59 million individual observations if there were no missing data. However the Treasury's estimated equation for the impact of EU membership involves only 390,521 observations. Since this is only 15% of the potential full dataset we assume that trade data for most countries and years is missing, or has zero values[5]. The only details of the Treasury's estimated equation are those given in table A.1 of their April 2016 Report (repeated below).

Table A.1: Results for goods analysis

Variable	Country-pair fixed effects
EU membership	0.766***
EU trade diversion	-0.035
FTA membership	0.219***
EEA membership	0.566***
GDP	1.085***
Population	-0.292***
Sample size	390,521

** Significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level*

Source: HM Treasury calculations

GDP and population enter in logs, coefficients on time dummies and country-pair fixed effects are not reported

The coefficients on the EU dummy variables are an average of EU members' deviations from the level of goods trade predicted by the underlying gravity relationship between trade and GDP, population and the country fixed effects. The coefficient on EU membership of 0.766 indicates that trade in goods (measured as an average of exports and imports) between EU members is raised by 115%[6]. The measured impacts of EEA or FTA membership are much lower, but in this paper we focus on the full EU membership impact as a worst-case scenario in which the UK reverts to trade on a WTO basis in 2019. The issue of trade diversion, i.e. EU members' loss of third party trade, is determined from the dummy EU1 where only one of a country pair is an EU member. The trade diversion coefficient is insignificant and the Treasury conclude that no significant trade diversion takes place on average for EU members.

There are several important short-comings in this approach to measuring the impact of EU membership on trade:

- The number of country pairs is very large, many of which are small developing nations undertaking minimal trade with the UK. The inclusion of such countries affects the underlying gravity equation and hence the EU2 dummy which is just an average deviation from the underlying equation. In technical terms the variance of trade is much higher for small countries than large countries and hence the measured errors are heteroscedastic.

- The Treasury measures the average impact of EU membership across all EU members, and does not investigate whether this calculated impact is relevant to the UK alone. When we replicate the Treasury analysis we find that there is a large and significant *negative* residual for the UK. Indeed, the Treasury Report provides virtually no information directly about UK trade with the EU. We will return to this issue below.

- The Treasury does not in all cases assume that the benefits to trade of EU membership are fully reversed on leaving[7]. The Treasury assumes that a lower scenario is possible in which only half of the benefits of EU membership are reversed within 15 years, since regulatory divergence may be slow. However, in the upper scenario they do assume a full reversal of benefits, despite the fact that UK firms must currently be compliant with EU regulations and are likely to maintain compliance to continue trading.

- The Treasury makes no allowance for changes in migration as a consequence of Brexit[8].

- Despite predicting a depreciation of sterling in its ‘Immediate Impact’ report, the Treasury makes no allowance for a long-term lower level of the sterling exchange rate which might offset higher costs imposed by EU tariffs[9].

OECD and IMF Gravity Equations

This paper focusses mainly on the Treasury gravity model estimates of the impact of Brexit but we also describe the less detailed approaches of the OECD and IMF.

OECD Equations

The OECD's assessment of the economic impact of Brexit parallels the Treasury in starting with trade and FDI and then estimating the consequent impact on productivity. An additional factor in the OECD analysis is to take account of potential changes in regulation, and of restrictions in migration, leading to lower investment in R&D and reduced managerial quality. Also like the Treasury, the OECD uses a gravity model to calculate the impacts on trade and FDI.

The OECD estimates that 'trade openness' will decline as a result of Brexit by between 10 and 20 per cent[10]. This is said to be based on an OECD gravity model paper by Fournier *et al.* but it is difficult to see how this paper supports these figures. The gravity model analysis covers only OECD members and a short time period of 1990-2012. The results are confusing and contradictory with some equations showing no rise in intra-EU exports as a result of EU membership. Other equations show a large increase (72%) in intra-EU trade. An average of zero and 72% would give 36% increase in exports due to EU membership. Reversing this gives a decline of (36/136=) 25% for exports to the EU or 11% for total exports, so it remains unclear why the OECD report adopts a range of 10-20%. The figure of 72% seems to us to be more plausible, but this is an average across all 28 EU members and there is no attempt to examine whether this applies specifically to the UK.

The gravity model equations of Fournier *et al*. are based on the theoretical gravity equation derived by Anderson & van Wincoop (see equation 4 below). However these gravity equations include country fixed effects terms only for countries and not for each trade-pair of countries as in the Treasury equation. Instead they include a set of specific dummies as outlined in equation 1 above. These include average GDP-weighted mean distances for each country in addition to the actual distance between the countries in each trade-pair. Without fixed effects dummies for each trade-pair this approach is not the one recommend by Head and Mayer (2015).

Three equations are estimated, in each case with data covering OECD countries and the period 1990-2011. Two of these are ordinary least squares equations, one with the coefficient on the GDP term set at unity and the other with this coefficient freely estimated. The third equation has a Poisson estimator[11], to deal with the problem of zero values of trade and heteroscedasticity. The estimates for the impact of EU membership, in each case, cover all EU members and not only those members which joined after 1990.

The IMF Gravity Equation

The IMF published its Brexit report in June 2016 shortly before the referendum[12]. The logic of the argument was similar to that of the OECD, with sections on trade, FDI, productivity and migration. Most of the report consisted of a selective literature review. The IMF had much less of its own research to use than had the OECD, but unsurprisingly had more to say on the UK financial sector.

The main contribution of its own research to the IMF report was a gravity model. Unlike any other Brexit report the IMF pointed out that the impact of EU membership on trade measured by most gravity models were averages across all EU member states. In an attempt to isolate the impact on the UK alone, the IMF conducted a gravity analysis with data confined to UK trade partners, over the short time-period of 2004-14. The shortness of the period made it more appropriate to use an analysis without fixed effects. In this case the measured impact of EU membership is simply the average difference between UK exports to EU members compared with UK exports to non-EU countries, after allowing for the size and distance from partner economies. The IMF estimates are that EU membership raises exports by 103% and services by 84%.

The IMF's conclusion on the long-term impact of Brexit was that '*The net long-run economic effects of leaving would … likely be negative and substantial, though there is significant uncertainty about the precise magnitude*'. The IMF was thus rather more tentative in its conclusions than the Treasury but clearly expected a large negative outcome on trade and hence on both GDP and per capita GDP.

The Theory Underlying the Gravity Model

The gravity model technique is, in our view, essentially empirical but Anderson (1979) and Anderson and van Wincoop (2004) have derived a 'theoretical' basis for the technique, described in a general setting in Head and Mayer (2015)[13]. This is widely referred to as the basis for understanding the technique. This theory does little more than combine the consumer theory of micro-economics with general equilibrium in trade to the gravity model and in practice adds little to a common-sense approach which would expect that there is more trade between larger countries (even if large countries are somewhat less open) and also more trade between countries that are closer together (reflecting transport costs) and have other features which increase or diminish the costs of penetrating markets.

The economic theory that derives a gravity equation, as exemplified by Anderson and Wincoop (2003) (A&W) introduces one new factor to the simple gravity equation in which bilateral trade is directly proportional to the product of each country's GDP and inversely proportional to distance. These authors add what A&W call 'multilateral trade resistance' (MTR) to the bilateral trade resistance such as distance between the two countries. MTR refers to a measure of the *average* trade barrier (including distance and tariffs) faced by any country in its trade with all other countries. In A&W, what matters is *relative* trade resistance, the trade resistance between any two countries compared with the resistance to trade with all other countries.

The theoretical structure of A&W is that each country produces a fixed supply of a single good. Consumer preferences are homothetic (with constant elasticity of substitution over all goods), i.e. the ratio of expenditures chosen depends only on relative prices and is independent of income or scale. There are supply prices which always fully adjust to ensure that all goods produced are purchased according to consumer preferences. Despite this general equilibrium framework, the gravity equation derived from this theoretical setup does not depend on relative prices or fixed supplies. The equation for bilateral trade x_{ij} between any country pair is a function of the product of each country's income, y_i and y_j, relative to world income, y^W,[14] the bilateral trade resistance between the country pair, t_{ij} and the inverse product of the multilateral trade resistance for each country, P_i and P_j.

$$x_{ij} = \frac{y_i y_j}{y^W}\left(\frac{t_{ij}}{\mathrm{P}_i P_j}\right)^{1-\sigma}$$

………………. (4)

where

$$P_j^{1-\sigma} = \sum_i P_i^{\sigma-1}\,\theta_i t_{ij}{}^{1-\sigma}$$

and

$$\theta_i = {}^{y_i}\!/_{y^W}$$

This structural gravity equation is a function of joint incomes (often implemented as GDP) and trade resistance terms (often implemented as distance, common border, common language and other variables). Relative prices are not included in the derived equation although they are an important feature of the theory from which the gravity equation is derived. It is as if the general equilibrium with flexible prices is extinct, leaving only a ghostly presence in the gravity model with a fossil in the form of the average degree of trade barrier/cost to add to the standard equation. The derivation is in terms of nominal values of trade and income, while the empirical implementation often uses nominal trade but real GDP.

One of the main claims made by A&W is that since theory predicts that multilateral trade resistance is a relevant determinant of bilateral trade, the absence of proxies for MTR in the estimation of gravity equations will result in omitted variable bias and hence incorrect inferences about effects such as membership of free trade areas.

Head and Mayer (2015) (H&M) define a similar structural gravity equation with MTR defined as an income or expenditure weighted sum of bilateral trade resistance terms. They show that this can be derived from a general equilibrium setup where all importers allocate expenditures across available goods and a market clearing assumption for all exporters. A&W's gravity equation can be considered a subset of H&M's structural gravity equation.

$$X_{ni} = \frac{Y_i}{\Omega_i}\frac{X_n}{\Phi_n}\phi_{ni}$$

where

$$\Phi_n = \sum_j \frac{\phi_{nj} Y_j}{\Omega_j}$$

and

$$\Omega_i = \sum_j \frac{\phi_{ji} X_j}{\Phi_j}$$

(using the notation of H&M). X_{ni} is the trade by exporter *i* to destination *n*, $\phi_{ni}, 0 \leq \phi_{ni} \leq 1$ represents bilateral trade costs, $Y_i = \sum_n X_{ni}$ is the value of production of country i , $X_n = \sum_i X_{ni}$ is the value of the importer's expenditure on imports from all source countries.

H&M note that estimation using pairwise fixed effects has become more common in many empirical applications in place of explicit dummy variables such as distance, common border etc. They state that if the derived gravity equation is in multiplicative form as in the above equations, the use of fixed effects will give consistent estimates of the trade resistance variables, which are usually the principal interest of researchers.

The Treasury notes in its literature review the papers by Anderson and van Wincoop (2003) and Head and May (2015) and justifies the specification of its gravity (equation 3 above) with the observation:

'In order to estimate a model that accounts for multilateral trade resistance, Anderson and van Wincoop (2004) suggest using country-pair fixed effects. This is supported by Head and Mayer (2013) who note that estimation of gravity models through econometric models with country fixed effects captures multilateral resistance terms and is therefore theory-consistent.' H. M. Treasury (2016), page 161, paragraph A.41, *op. cit*[15].

In the Treasury analysis, the treatment of any MTR effects, other than the impact of FTAs, is absorbed into the fixed effects as factors that do not change over time.

Alternative Forms of the Treasury Gravity Model

Because the Treasury's estimates of the impact of Brexit seem high, and the methods used are opaque to most policy makers and to the general public, we have constructed a database and attempted to replicate the gravity model analyses of the Treasury, OECD and IMF for trade in goods.

Data Sources

Our trade data is from the IMF's Direction of Trade Statistics for 1950-2015 in current dollars. This is the same source as the Treasury's. GDP and population data is from the Conference Board, with GDP in 1990 dollars at purchasing power parity. Distance between trade partners is from the CEPII's GeoDist database. Dummy variables are constructed.

Gravity Models with Fixed Effects

Our initial gravity model equations take the same form as those in the Treasury report, including the use of fixed effects but we also examine other forms of the gravity equation for comparison.[16] Importantly, we are able to closely reproduce the Treasury estimate of an uplift of 115% on trade between EU members relative to what is predicted by an underlying gravity equation. The measure of trade used by the Treasury was an average of exports and imports, whereas our dependent variable is exports in current dollars from one country to another where data is available. This difference in dependent variable might make a small difference to results but is unlikely to be substantial. Zero values for exports are omitted from the equations with fixed effects[17]. Importantly, we notice that most gravity equations tend to over-predict the level of UK exports to EU countries, which the Treasury report did not comment upon. We return to this important issue below.

We have also estimated gravity model equations for more recent periods than the long 1948-2013 period in the Treasury report. The Treasury's full-period equation for 1948-2013 estimates the average gain to intra-EU trade among all EU members joining since the foundation of the EU in 1958. Shorter periods capture the impact on trade for those countries which join the EU during the period. The full post-war period is long and it may be that the advantage of EU membership has changed over time. One obvious reason is that international tariff barriers have declined over time. The various multilateral trade agreements under the General Agreement on Trade and Tariffs (GATT), the World Trade Organisation (WTO) and the General Agreement on Trade in Services (GATS) have reduced international tariffs and non-tariff barriers. The contrast in tariff barriers between the EU and the rest of world is now smaller than it was in the 1950s and 1960s[18]. Gravity models may allow for this change over time, along with average global price inflation, and factors such as world recessions but this is unclear.

Our first equation is estimated by an ordinary least squares (OLS) equation with time and country pairwise fixed effects and robust variances, fitted across all countries in the sample. The equation fitted over the full 1950-2015 period is shown in annex B, table B.1. The coefficients for this period are summarised in the first column of table 1 below. The equation is similar to that of the Treasury except that the Treasury's EEA dummy is here replaced by two dummy variables respectively for Norway's and Switzerland's exports to the EU. Also, a dummy variable is added for trade-pairs where both countries are members of an FTA other than the EU including NAFTA.

The equations for successively shorter and more recent periods are summarised in the other columns of table 1. The values are similar to other gravity model estimates, with coefficients on GDP generally close to unity as expected. However, we should note that in the periods beginning in 1950 and 1960 the coefficients on exporter's GDP are much smaller. In these periods it is per capita GDP which is more important. Estimates of the average trade advantage due to EU membership (EU2, the 'EU effect') diminish over time, as shown in

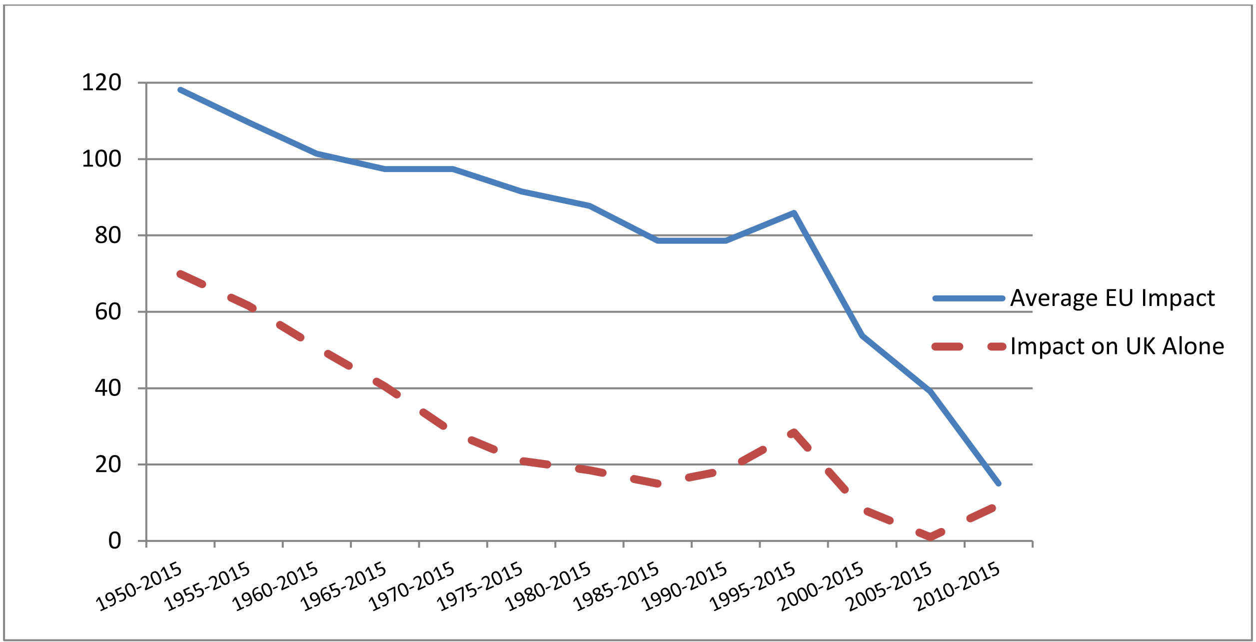
Table 1 Gravity Model Equation Coefficients (ordinary least squares, fixed effects, 120 countries)

	1950-2015	1960-2015	1970-2015	1980-2015	1990-2015	2000-2015	1950-2000
Constant	-4.824	-6.377	-9.793	-10.363	-11.271	-10.815	-2.360
Ln(GDPi)	0.523	0.602	0.909	1.131	1.187	0.952	0.244
Ln(GDPj)	0.949	1.031	1.073	0.896	0.931	1.064	0.961
Ln(GDPi/POPi)	1.065	0.978	0.632	0.377	0.245	0.432	1.403
Ln(GDPj/POPj)	0.174	0.090	0.094	0.344	0.361	0.429	0.220
UK_EU	**-0.237**	**-0.280**	**-0.408**	**-0.438**	**-0.385**	**-0.363**	**-0.153**
EU2	**0.779**	**0.686**	**0.663**	**0.604**	**0.556**	**0.438**	**0.949**
$EU1_i$	0.152	0.136	0.159	0.165	0.249	0.261	0.175
$EU1_j$	-0.136	-0.187	-0.159	-0.173	-0.156		
FTA	0.390	0.316	0.327	0.315			0.637
Norway	0.448	0.412	0.332	0.443	0.378	0.225	0.282
Switz	0.755	0.590	0.377	0.282	0.175		0.784
R2	0.842	0.841	0.849	0.857	0.879	0.906	0.847
Observations	459211	433526	397554	348002	284589	191953	278362

Note: The dependent variable is the log of trade in current dollars between all trade pairs. All values are significant at the 1% level except for those in italics (5%). Values not significant at the 5% level were omitted from the equations.

the upper line in chart 1. The implication is that the EU effect is lower for those EU members which joined the EU later in the post-war period. For instance the impact measured in the 1995-2015 period includes only the uplift in trade for the Eastern European countries, Malta and Cyprus which joined the EU in 2004, plus Romania and Bulgaria (2007) and Croatia (2007)[19]. There is some rise in the measured impact in the 1995-2015 period which may reflect the influence of the setting-up of the Single Market under the Maastrict Treaty of 1992. The sharp fall in the following periods is unexpected since the impact on Eastern European trade is likely to be large, and is estimated as large by Reuven Glick of the Federal Reserve Board of San Francisco[20]. One factor may be the role of the Euro in slowing economic growth and hence demand for imports within and

Chart 1 The Uplift in Intra-EU Trade due to EU Membership (percent)[21]



Note: these results were generated from gravity equations estimates with fixed effects on a dataset including all 120 countries.

around the Eurozone, but it not obvious to us that a gravity model is able to identify growth effects like this[22].

The final column in table 1 is for the period 1950-2000 in which the only EU joiners were the EU15 countries of Western Europe. In this case the coefficient on the EU2 variable is high at 0.95 indication an average uplift of 161% in exports due to both countries in a trade-pair being EU members.

Impact of EU Membership on the UK Alone

As the IMF point out for gravity models used to estimate the impact of EU membership on trade:[23] '*the estimated coefficients are therefore in a broad sense, averages across all EU economies*'. This raises the question of whether they can be assumed to apply to the UK. The IMF tests this issue by estimating a gravity model without country fixed effects for UK trade data alone for the short period 2004-2014. The IMF calculates that EU membership for the UK doubled trade with EU partners and hence confirms the Treasury's estimate.

We have replicated this analysis with our IMF trade data for goods (the IMF paradoxically use UK data from ONS) and get lower estimates for the impact of EU membership. Depending on the exact approach used we estimate an impact of either 8% or 68%[24]. The methods used by the IMF are not directly comparable with the Treasury and the comparison with our own analysis suggests that a wide range of estimates is possible depending on data sources and on the precise methods used. However, as expected, all of the estimates show a positive, if not always significant, impact of EU membership on trade.

We have approached the issue of a UK-specific impact of EU membership in a different way which allows us to maintain comparability by continuing to use a fixed-effects approach. Instead of a UK-only data set we maintain the full dataset of 120 counties and estimate the average impact on trade for all 28 EU members in the same way as above. The degree of under-prediction for the UK alone is measured by including a new dummy variable (UK_EU) which measures the average deviation of actual UK exports from that predicted by the equation. This variable indicates that UK exports to EU countries are around 30% lower than the average level of intra-EU exports for the new EU members in each period (table 1). In using the average value for all EU members as if it applied to the UK as much as to other EU members, the Treasury fails to take account of the fact that UK exports to the EU are over-predicted (whereas German, French or Italian exports to the EU are not).

To calculate the impact of EU membership specifically on the UK we add this (negative) value to the calculated average value for EU membership across all EU members. The resulting values are shown in the 'impact on UK alone' in chart 1. The impact of EU membership on UK goods exports obtained in this way from an equation estimated over the whole period, 1950-2015, is 70% rather than the 115% in the Treasury report. Once again this estimate is smaller in equations estimated over shorter, more recent, periods, and declines to around 10% in equations fitted from 2000-2015. In these recent periods the average EU impact relates to countries which joined the EU during the period, and the impact for the UK alone is thus relative to the average for these joining countries.

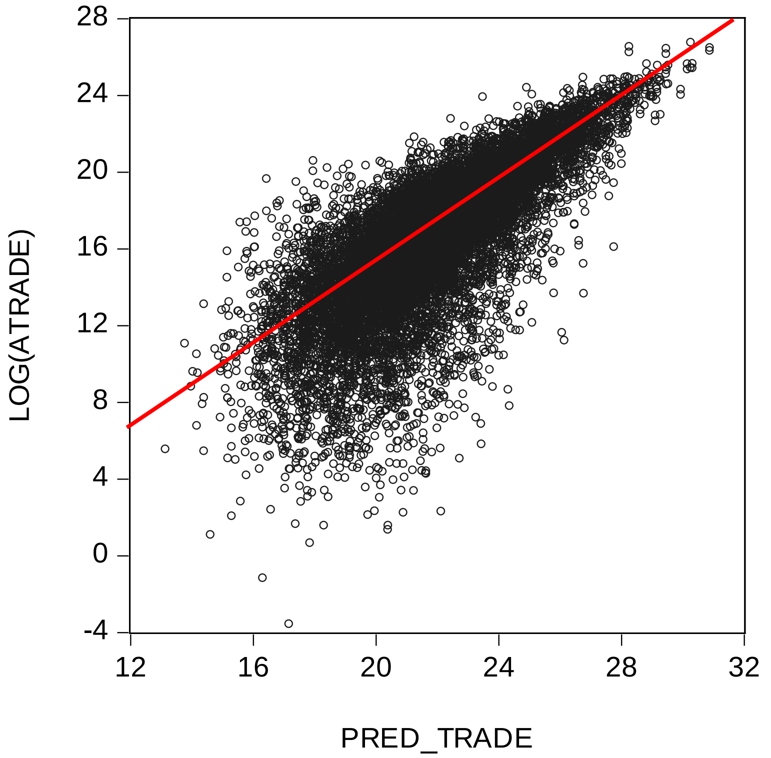
For the period 1950-2000, when only western European countries were members of the EU, the coefficient on the EU2 dummy is 0.95. The coefficient on the UK_EU dummy (UK exports to EU countries) is small at -0.15 (table 1, column 7). The sum of these two coefficients gives a large value of 0.8 indicating an uplift in UK exports of 125%. This accords with the large increase in the share of UK exports going to the EU through most of this period, even though this was mainly a continuation of the pre-1973 trend in this export share. After the 1990s this share began falling and is now close to the 1973 level[25].

Estimates based on Trade with a Subset of Countries

In a gravity equation estimated with data on the trade of over 100 countries with each other (more than 10,000 trade-pairs), over a 65 year period, a significant number of the recorded trade flows are miniscule. Each trade observation counts as one data point whether it be trade between Azerbaijan and Angola, or between the USA and China. This can be seen for 2015 on chart 2 which plots the level of trade in 2015 for all trade-pairs against the predicted level, which depends on GDP, population and distance apart. The scatter of very low trade observations in the lower left quadrant of the chart, mainly for small countries, generates a problem of heteroscedasticity. This would be even more visually obvious if we were to include all observations since 1950.

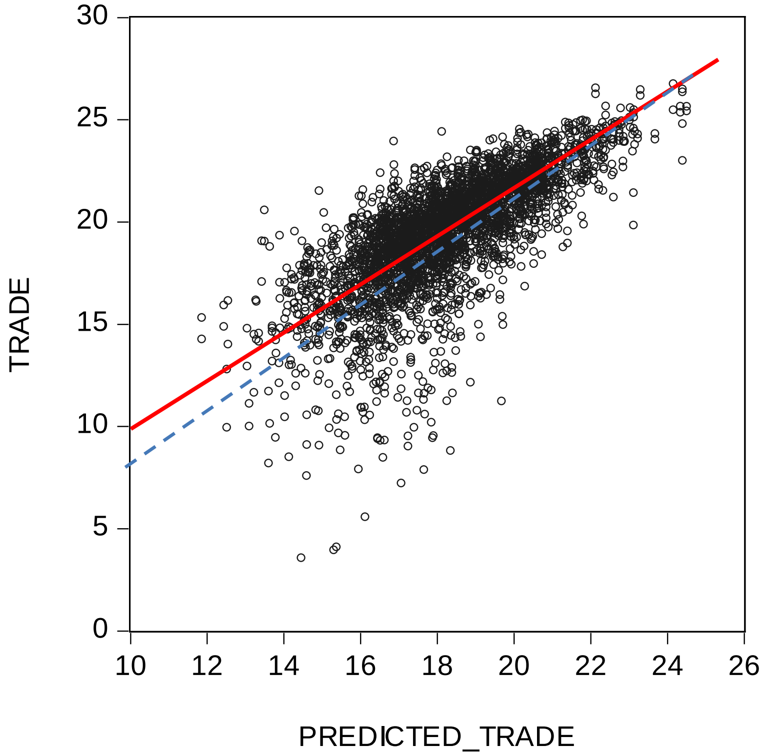
The importance of this for measuring the impact of EU membership is that the inclusion of these countries affects the underlying gravity equation, and hence also the EU impact measure which is just an average deviation from the underlying relationship. In practice these outliers contribute to generating a lower, steeper line on chart 2 than would otherwise be the case. Since trade observations for trade-pairs with two EU members are generally above the line in chart 2, the average distance from line for these EU trade-pairs is increased.

Chart 2 Trade in 2015 Between the All Export Partners ($ logged)



Note: Predicted trade is taken from a gravity equation with terms in GDP, per capita GDP and distance, without country fixed effects.

Chart 3 Trade in 2015 Between the Top-60 Export Partners of The UK ($ logged)



To minimise the influence of countries with small trade flows of little relevance to the UK we have repeated the analysis using the same equation but restricting the data to only the UK's top 60 export markets (illustrated in chart 3). This trade accounts for close to 100% of all UK exports and includes all 28 EU member states. There are 3500 trade-pairs in this dataset compared with 13,100 in the full dataset, but this is the set of countries of most relevance to the UK's trade. The solid line on chart 3 is the best-fit line through this data. For comparison we he have also included a pecked line which represents the best-fit line through all of the data for the full 120 countries from chart 2. The pecked line starts from a lower part and rises more steeply than the solid line because there are more small-trade outliers in the fill sample. This reduces the estimate for the impact of EU membership.

Table 2 Gravity Model Equation Coefficients (OLS, fixed effects, Top 60 UK Export Markets)

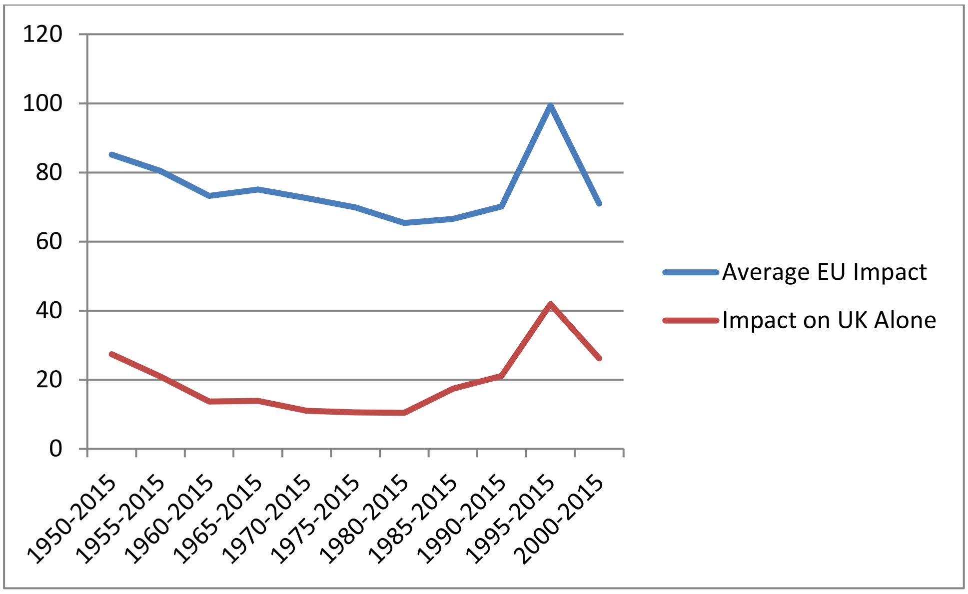
	1950-2015	1960-2015	1970-2015	1980-2015	1990-2015	2000-2015	1950-2000
Constant	-5.994	-8.258	-10.725	-10.467	-17.766	-16.690	-2.549
Ln(GDPi)	0.699	0.800	0.982	1.244	1.640	1.918	0.326
Ln(GDPj)	0.992	1.119	1.164	0.822	1.151	*0.786*	1.007
Ln(GDPi/POP)	0.889	0.781	0.616	0.406		-0.499	1.284
Ln(GDPj/POP)	-0.057	-0.173	-0.158	0.316		0.485	-0.088
UK_EU	**-0.374**	**-0.421**	**-0.441**	**-0.404**	**-0.340**	**-0.304**	**-0.285**
EU2	**0.616**	**0.549**	**0.545**	**0.503**	**0.532**	**0.537**	**0.763**
$EU1_i$	0.153	0.129	0.156	0.177	0.252	0.370	0.170
$EU1_j$	-0.089	-0.111	-0.059	-0.057		0.175	
FTA	0.586	0.551	0.562	0.574	0.434		0.575
Norway	0.169	0.128					
Switz	0.499	0.354	0.156				0.552
R2	0.884	0.886	0.887	0.895	0.913	0.937	0.880
Observations	165860	152855	138157	114048	88592	56988	112375

Note: The dependent variable is the log of trade in current dollars between all trade pairs. All values are significant at the 1% level except for those in italics (5%). Values not significant at the 5% level were omitted from the equations.

If we were also to remove all of the cases on chart 3 with annual trade of less than one million dollars, the new best-fit line would start at an even higher point and then rise even more slowly than in chart 3. This would further reduce the average deviation of the intra-EU trade from the best-fit line and hence reduce the estimate of the impact of EU membership.

The results from the equation estimated across the top-60 UK export markets are shown in table 2. The average impact of EU membership, as measured by the EU2 dummy, is smaller than for the all-data case in table 1 estimated over the whole 1950-2015 period (table 2, column 1). This EU impact is also more consistent over the successive periods, averaging around 0.5 which indicates an uplift in trade of 69%, compared with the Treasury's 115%. The average impact of EU membership on intra-EU trade, estimated from this 'Top 60' sample, is shown in chart 4. A comparison of this with the impact estimated using all countries is shown in chart 5. The latter chart shows that the average EU impact is a little lower than obtained using the full sample of countries, at around 80-90% for the longest time period.

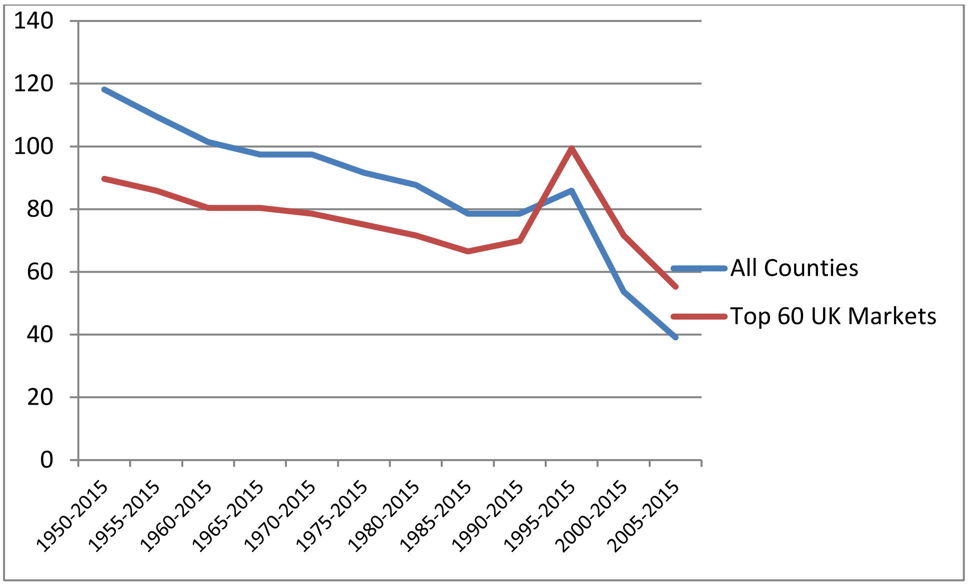
Chart 4 The Percentage Uplift in Intra-EU Trade due to EU Membership estimated from the Top 60 markets for UK exports



Note: these results were generated from gravity equations estimates with fixed effects on a dataset including all 120 countries.

The UK_EU dummy, for UK exports to the EU, is also relatively consistent at close to -0.37. The resulting impact for the UK alone is much smaller over the whole period than when measured with the full set of countries. Equations estimated over periods since 1970, or more recent periods, result in an estimate for the UK alone generally in a range of 10-30%. This is not dissimilar to the range obtained using the full 120-country dataset shown in chart 3 above.

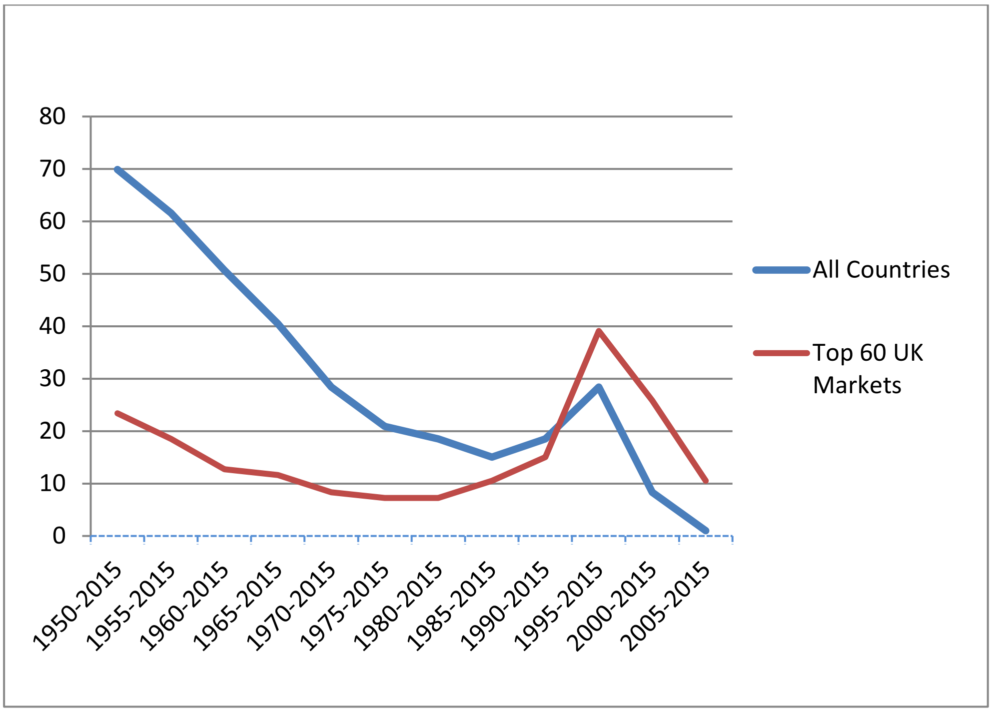
Chart 5 Average Percentage Increase in Trade due to EU Membership



Note: these results were generated from gravity equations estimates with fixed effects on a datasets including all 120 countries, and the Top 60 UK export markets respectively.

The range of estimates for the UK alone, are shown in chart 6 for both the full set of countries and for the Top 60 dataset. Since the Top 60 dataset is most relevant to the UK and includes a wider range of trade between non-UK countries than the smaller samples, we prefer this as an estimate of the impact of EU membership on UK exports. These 'Top 60' estimates are also the most consistent over time, and again show the largest increase after the Single Market is formed. The uplift from EU membership for UK exports varies between 10-40% and averages around 20%. This is much lower than the estimated impact on exports for all EU members. A full reversal of this level of EU impact would give a reduction in exports of (20/120=) 16% and a range of 9-29%. This is lower than the estimated impact on exports for all EU members, and much lower than the Treasury estimate of a 115% uplift.

Chart 6 Increases in Exports (%) due to EU Membership for the UK Alone



Note: these results were generated from gravity equations estimates with fixed effects on a datasets including all 120 countries, and the Top 60 UK export markets respectively.

Estimates without Fixed Effects

As a second examination of dealing with the problem of heteroscedasticity resulting from low trade volumes among small emerging economies, we generate gravity equations without fixed effects using a Poisson quasi-maximum likelihood estimator[26]. In this case the fixed effects for individual countries are replaced by a range of dummy variables for distance, contiguity of borders, and common languages, membership of free-trade areas and separately for membership of the EU. The distance variable is also expressed relative to the mean distance for each country.

$$\ln X_{ijt} = \alpha_{ij} + \gamma_t + \alpha_1 \ln(Y_{it}Y_{jt}) + \alpha_2 \ln\left(\frac{Y_{it}}{POP_{it}}\right) + \alpha_3 \ln\left(\frac{Y_{jt}}{POP_{jt}}\right) + \ln\left(\frac{DIST_{ij}}{R_i R_j}\right) + CONTIG_{ij} + \beta_1 EU2_{ijt} + \beta_2 EU1_{ijt} + \beta_3 CHINESE_{ijt} + \beta_4 GERMAN_{ijt} + \varepsilon_{ijt}$$

where:

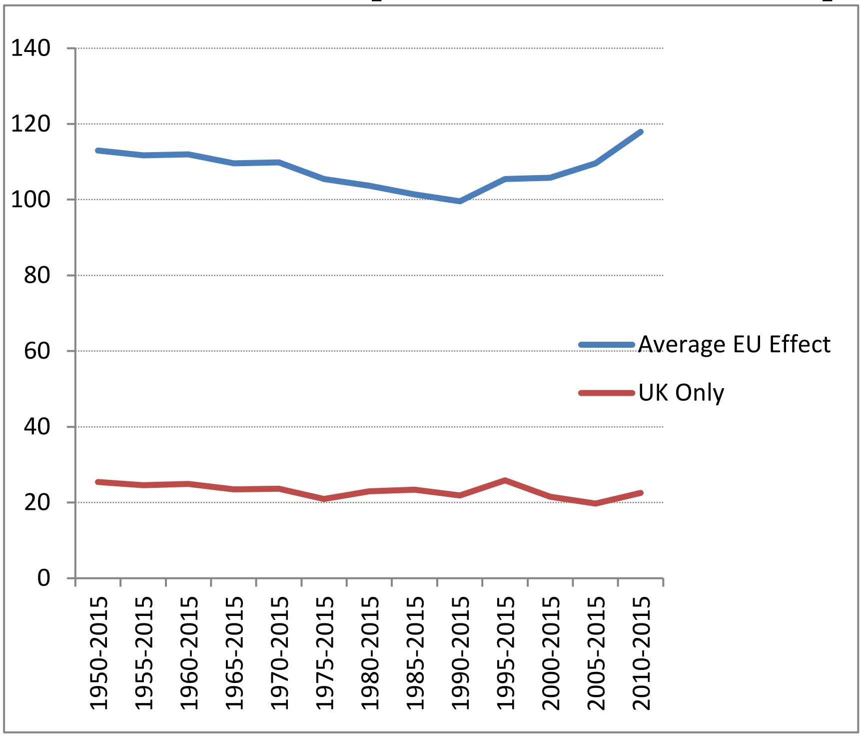
- R is the mean distance from each country to all others
- Chinese is where both countries speak Chinese
- German is where both countries speak German

Table 3 Gravity Model Equation Coefficients (Poisson quasi-Maximum likelihood, All Countries)

	1950-2015	1960-2015	1970-2015	1980-2015	1990-2015	2000-2015	2010-2015	1950-2000
Constant	-12.963	-12.928	-12.771	-12.546	-12.421	-12.195	-11.640	-13.503
Ln(GDPi)	0.851	0.850	0.845	0.837	0.831	0.830	0.835	0.858
Ln(GDPj)	0.862	0.861	0.857	0.849	0.844	0.843	0.827	0.876
Ln(GDPi/POPi)	0.450	0.448	0.440	0.435	0.428	0.402	0.418	0.517
Ln(GDPj/POPj)	0.447	0.446	0.439	0.439	0.441	0.429	0.407	0.443
Ln(Dist/(Ri*Rj)	-0.819	-0.821	-0.824	-0.828	-0.835	-0.830	-0.793	-0.811
CONTIG	0.109	0.112	0.124	0.158	0.202	0.222	0.240	
UK_EU	**-0.530**	**-0.529**	**-0.529**	**-0.505**	**-0.493**	**-0.527**	**-0.576**	**-0.375**
EU2	**0.756**	**0.751**	**0.741**	**0.712**	**0.691**	**0.722**	**0.779**	**0.677**
FTA	0.217	0.212	0.197	0.149	*0.091*	*0.100*	*0.184*	0.362
Norway	0.506	0.502	0.496	0.472	0.457	0.523	0.608	0.379
Switz	0.318	0.312	0.312	0.300	0.293	0.333	0.502	0.247
Chinese	2.580	2.573	2.546	2.488	2.402	2.299	2.168	3.581
German	0.497	0.498	0.500	0.494	0.478	0.493	0.457	0.493
Observations	722336	627221	531552	435802	332536	213617	81520	521674

The estimated equation for all countries over the full period is shown in annex B.3. The variable coefficients are shown in table 3 and the EU impacts are summarised in chart 7. In this case, without country fixed effects terms, the EU effect (EU2) refers to all EU members in each period and not solely to the new members joining within the period.

Chart 7 Increase in Exports due to EU Membership for the UK Alone (%)



Note: these results were generated from gravity equations estimates without fixed effects on a datasets including all 120 countries.

The average impact of EU membership on exports for all EU members is almost exactly the same as the Treasury's 115% when measured over the whole 1950-2015 period (chart 7). This impact tends to fall slightly for more recent time periods until 1990 when it begins to rise again, reaching close to 120% in the 2010-15 period. For the period 1950-2000 (i.e. excluding the new members in 2004 and subsequent years) the estimated impact is a little under 100%.

Once again the key point is that the estimated impact on UK exports to the EU is much lower, in the range of 20-25%. This is similar to the range shown for fixed effects equations in chart 6. In both cases the estimates are very much lower than the 115% figure in the Treasury report. Reversing a 20-25% increase in exports would lead to a 16-20% loss of UK exports to the EU. In the Treasury's case a reversal of the 115% increase would result in a loss of trade with EU of 45%. Our estimate of the loss is thus under half of that published by the Treasury.

It is also possible to estimate the gravity equation with data restricted to the UK's top 60 export markets, although this should be less necessary using a PPML estimator. The results of using this Top-60 data are set out in table 4. The average EU-effect across all EU members was 100% for the period 1950-2000, i.e. before the accession of the new A10 members[27]. The value for the UK alone was 35% for the same period, but essentially negative for the period since 2000-2015.[28]

Table 4 Gravity Model Equation Coefficients Without fixed effects Top 60 UK Export Markets)

	1950-2015	**1960-2015**	**1970-2015**	**1980-2015**	**1990-2015**	**2000-2015**	**2010-2015**	**1950-2000**
Constant	-12.275	-12.223	-11.964	-11.593	-11.258	-10.848	-10.347	-12.878
Ln(GDPi)	0.826	0.824	0.818	0.805	0.793	0.786	0.791	0.829
Ln(GDPj)	0.853	0.852	0.845	0.834	0.824	0.818	0.801	0.860
Ln(GDPi/POPi)	0.468	0.466	0.453	0.441	0.425	0.394	0.409	0.546
Ln(GDPj/POPj)	0.495	0.493	0.482	0.475	0.469	0.455	0.436	0.487
Ln(Dist/(Ri*Rj)	-0.777	-0.779	-0.781	-0.788	-0.799	-0.795	-0.765	-0.780
CONTIG	0.122	0.125	0.138	0.166	0.211	0.238	0.251	
UK_EU	**-0.606**	**-0.605**	**-0.604**	**-0.585**	**-0.579**	**-0.625**	**-0.728**	**-0.404**
EU2	**0.688**	**0.683**	**0.669**	**0.629**	**0.580**	**0.587**	**0.635**	**0.697**
EU1i	-0.065	-0.067	-0.065	-0.069	-0.074	-0.085	-0.083	
FTA	0.248	0.244	0.231	0.178	*0.096*	*0.089*	0.144	0.492
Norway	0.482	0.477	0.466	0.426	0.379	0.425	0.498	0.388
Switz	0.285	0.279	0.273	0.247	0.206	0.221	0.381	0.260
Chinese	2.616	2.608	2.574	2.499	2.377	2.250	2.114	3.726
Portugese	0.535	0.527	0.496	0.470	0.454	0.502	0.504	
German	0.578	0.580	0.583	0.581	0.571	0.597	0.559	0.552
Observations	22255	193181	162481	130661	94796	58336	21876	167875

Trade Diversion

A separate important issue in measuring the impact of EU membership on trade concerns trade diversion. It is possible that membership of the EU increases trade with other EU members at the expense of trade with non-members. To measure this, the Treasury added a dummy variable into the gravity equation to identify pairs of countries in which one trade partner is an EU member and the other is not. Using this approach the Treasury reports no significant effect, i.e. there was no evidence that increased trade with other EU members was associated with diminished trade flows with non-members.

We follow a similar procedure, but since our data is for exports rather than for two-way trade our dummy variable identifies trade-pairs in which the exporter is an EU member and the importer is outside the EU. Using a conventional gravity equation with country and time fixed effects the results suggest a win-win situation in which trade with non-members is higher once countries become members of the EU. The impact is also higher for newer members of the EU.

In contrast, equations without country fixed effects, but including a range of dummy variables for common languages, contiguous borders etc., show either no significant trade diversion, or in the case of the top-60 dataset a significant negative impact. Moreover the degree of trade diversion increases as the sample size increases. We have therefore been unable to generate a clear answer on the question of whether trade diversion takes place. However we adopt the equation without fixed effects, based on the all-country dataset, as our preferred equation since it generates the most plausible results. In this case the export diversion is not significant (and is excluded from equation B3of the Annex). The conclusion on export diversion does however depend on the precise specification of the gravity model.

Service sector trade

The Treasury report used a similar gravity model approach to estimate the impact of EU membership on trade in services. The data includes a large range of countries over the period 1981-2009. Once again the method found a positive impact of EU membership, albeit smaller than for goods, and again no evidence of trade diversion. The increase in intra-EU trade due to membership of the EU was estimated as 24%.[29] We have not re-created the data and equations for services trade and hence use the Treasury estimate to calculate the impact of EU membership on the aggregate trade in goods and services.

FDI, Productivity and GDP

The main focus of this working paper is the gravity model in the context of trade. For completeness in this section we add a few comments on the remainder of the Treasury's methods in estimating the impact of Brexit on GDP. These are the influence of Brexit on foreign direct investment (FDI), the knock-on impacts of lower trade and FDI on productivity, the impact of lower migration and the resulting impact of all of these on GDP, incomes and unemployment calculated via a macro-economic model.

The Treasury Report's analysis of the impact of Brexit on foreign direct investment (FDI) begins by using an estimate from Bruno *et al.* (2016) that FDI inflows into the UK from the EU would be 22% lower if the UK left the EU for a WTO rules based trade regime. The Treasury then assumes, with little evidence, that FDI flows from outside the EU would fall by a similar amount. If we follow the Treasury and take the estimate of a 22% fall in the flow of FDI from all sources, our calculations suggest that this scale of reduction in the *inflow* would be associated with about a 1% p.a. decline in the *stock* of FDI. The Treasury uses a sectoral production-function approach to calculate that each 1% reduction in the stock of FDI in the UK would, in turn, reduce productivity in the UK by 0.04%. Hence a decade after leaving the EU it would reduce the level of productivity by 0.4%, which is a small effect.

These calculations for FDI are complicated by the fact that FDI is measured as financial flows, and these are dominated by mergers and acquisitions and by financing flows. The latter are known to be influenced by taxation considerations leading to large inflows into tax havens including Luxemburg, Ireland and the Netherlands. The OECD estimate that on average only one third of FDI flows consist of physical investment. Our estimate for the UK is around one quarter. FDI in new productive activities is likely to raise productivity. Mergers and acquisitions may or may not do so. Mergers like the Kraft take-over of Cadbury which result in plant closures and the removal of the HQ to a tax haven, have a less obvious positive impact on productivity. In such cases productivity may be raised at the cost of lower levels of activity. For these reasons we feel that the potential impact of leaving the EU on productivity via the FDI channel is particularly uncertain. However, since survey evidence suggests that multi-national companies value the UK's membership of the EU in making decisions to invest in the UK, we assume that leaving the EU single market will have some detrimental impact on physical FDI.

We can note in passing, that the Treasury's citing of Canada as an example of the gains to productivity from joining an FTA are less than convincing. HMT cited a paper by Melitz and Trefler (2012) which shows that productivity in Canadian manufacturing rose by 14% in a few years after joining the US-Canada FTA in 1989. Part of this was due to the closure of low productivity plants and part due to rising productivity within survivors. An examination of change in per capita GDP (at PPP) in the total economy however shows an immediate fall in per capita GDP and a failure to regain Canada's pre-1989 trend for 20 years. Canada's per capita GDP also fell relative to the USA and remained low. What may have happened is that labour which was displaced through increased competition within the FTA was not re-employed at equivalent wages for decades.

More generally the Treasury assume that a 1% increase in the UK's trade openness, i.e. the ratio of trade to GDP, leads to a 0.2-0.3% increase in UK productivity. This estimate is derived from a number of studies, chiefly by Feyrer (2008), which conduct gravity-model analyses across a wide number of countries[30]. These countries are dominated by small emerging economies and it is not clear whether the conclusions apply to relatively small trade changes in an advanced economy. Even if we adopt the Treasury assumption, the impact would, in our view, be small. Our predicted decline in export openness following a Brexit move to WTO rules, would peak at 3% in 2023 and recover to 1% by 2030. Any decline in productivity is thus likely to be small.

The Treasury makes no assumptions about migration policy post-Brexit and instead uses the ONS population projections in all of its scenarios. The ONS arbitrarily assumes a drop in migration after 2019 from around 330,000 per annum to 185,000. The OECD assumes a fall in migration of 116,000 in their pessimistic scenario. In both cases the consequent slower growth in the labour force results in lower GDP. Both mention the possibility of loss of skills, but in practice controls on skilled migration are, in our view, less likely. The IMF argues that the form of migration controls is important and that it is thus not easy to calculate the impact of lower migration. Nevertheless, the IMF suggests that restrictions on migration are likely to damage labour supply reducing GDP, and potentially damage skills levels and efficiency leading to lower GDP per head.

These various calculated impacts of Brexit on trade, FDI, productivity etc., are finally converted into macro-economic aggregates to predict overall impacts on GDP, incomes and unemployment. Both the Treasury and the OECD feed their estimates into NIESR's NiGEM model. In both cases monetary policies and exchange rates are held constant. The IMF uses its staff resources to calculate aggregate impacts over a short five-year horizon but the methods are not described. The NiGem model is a multi-national general equilibrium system which uses CES production functions to govern demand for the factors of production and a price system to bring demand into balance with supply.

The mid-range estimates of the reduction in GDP in 2030 under a WTO scenario are 7.2% for the Treasury and 5.1% for the OECD. The IMF makes quantified predictions only up to 2021 but expect a negative long-term impact for Brexit under a WTO scenario. In our case we have entered our own estimates for impacts on trade and FDI into our Keynesian structural econometric model of the UK economy[31]. Unlike the Treasury or OECD we have not assumed unchanged monetary policy and exchange rates or unchanged fiscal policy. Instead exchange rates are determined by equations and adapt to the changing circumstances of Brexit. Short-term interest rates are exogenous and we assume that the Bank of England continues to adopt low rates to offset any danger of negative expectations resulting from the Brexit referendum results. Government fiscal policy is a little relaxed from the pre-referendum period in line with the plans announced in the 2017 Budget, but from 2019 is relaxed further to avoid rising unemployment. The impact of a move to WTO rules in 2019 is modelled as an increase in costs for both exports and imports. The increase is set to generate what would be a 10% decline in exports if projected on a constant exchange-rate basis. The actual decline in exports is around 7% because the lower exchange rate since mid-2016 mitigates the cost increases due to higher tariffs and administrative costs. The consequence of these assumptions is a small reduction in GDP by 2025 relative to a non-Brexit baseline (1.5%). The assumed fall in migration of 115,000 by 2025 contributes to the fall in GDP but also has the effect that per capita GDP remains little different from the baseline throughout the period. The monetary and fiscal policies required to generate these results prove to be sustainable in both inflation and government deficits and debt.

Conclusions

We conclude that gravity models generate estimates of the impact of EU membership on exports which are variable but for all EU members are always positive and significant. The Treasury's review of the gravity model literature also found a wide variety of estimates for the average impact of EU membership on trade in both goods and services. Their table A.5 is reproduced below.

HM Treasury Table A.5 External and HM Treasury estimate of EU and FTA membership effects

	EU Membership Effect		FTA Membership Effect
HM Treasury	68%/76%/85%		14%/17%/21%
OECD(2015)	60%		N/A
Baier,Bergstrand et al (2008)	92%		58%
Hufbauer and Schott(2007)	31%		27%
Carrere(2006)	104%		N/A
Eicher and Henn(2011)	37%		Insignificant
Eicher et al(2012)	51%		N/A

The range of impacts for the HM Treasury results is based on using a +/_1 standard error range

All of our alternative estimates for the average EU effect across all member states are lower than that of the Treasury. However the range of estimates still approximates to a doubling of export trade inside the EU.

This average impact however does not relate directly to UK trade. UK exports of goods to the EU have usually been well below the levels predicted by these gravity model equations. Instead of the Treasury's average impact across all 28 EU member states of around 115%, the increase in the UK alone appears to be in the range 20-25%.[32] There is evidence that the impact on UK exports was somewhat higher than this before 2000, which accords with the evidence from time series trends showing that the share of UK exports (of goods and services) rose rapidly from accession in 1973 to 1990. Since 1990 the share first stalled and in recent years has been falling rapidly as non-EU markets have grown much faster than those within the EU and within the Eurozone.

These estimates for the impact of EU membership on trade in goods may or may not be reversed when the UK leaves the EU in 2019. A 25% uplift in exports due to EU membership, would, if fully reversed, equate to a 20% loss of exports. However, not all of the gains are likely to be reversed, and especially not immediately. While tariffs on goods may be imposed overnight if the UK reverts to WTO rules, and administrative costs imposed if the UK is outside the customs union, non-tariff barriers may be initially low since UK firms are mostly already compliant with EU regulations. The Treasury takes this latter point into account in its low and medium projections but not for its high projection.

Our estimates of the impact on the UK alone are close to the impact of WTO tariffs given in the ESRI study, cited earlier. The ESRI calculated that UK trade in goods with the EU might fall by 22% if the UK adopted WTO rules. Estimates obtained using gravity model equations implicitly include the impact of higher administrative costs for borders and the impact of regulatory differences. The former will be relevant outside the EU customs unions, but the latter should be largely absent at least in the early years. The ESRI's estimate of the impact solely of tariffs may thus be nearer to the true impact than any estimate based on a gravity model. However, neither the gravity model nor the ESRI's more direct approach take into account the post-referendum depreciation of sterling. This depreciation has been large enough to offset the impact of higher tariffs on any loss of trade in around 90% of commodities.

We estimate that the overall impact of a Brexit involving a move to WTO rules would be relatively small. One important influence in making this calculation is the depreciation in sterling which occurred immediately after the Referendum. We expect the depreciation to be maintained well into the next decade, with only a very slow recovery in the real exchange rate. While a lower value for sterling assists the growth of exports and GDP, it also leads to higher inflation peaking at 1.4% p.a. higher in 2019 than in our pre-Referendum scenario. The extent of higher inflation depends on what one assumes about the tightness or looseness of monetary policy in the two scenarios. A second important influence is that we expect fiscal and monetary policy to help in generating this relatively benign outcome, with short-term interest rates at 1.8 points below the baseline level in 2017 but converging again after 2019. Some sustainable fiscal expansion is also assumed from 2019. The conclusions are firstly that the impact of Brexit on UK trade will be smaller than estimated by the Treasury, and secondly that negative impacts can be substantially offset by sustainably accommodating monetary and fiscal policy.

The macro-economic projections in this paper were generated with the May_2017_WP490 version of the CBR UKMOD macro-economic model.

Notes

1 H M Treasury Analysis. The Long-term economic impact of EU Membership and the Alternatives. April 2016 Table A.5. Cmnd 9250. OECD. The Economic Consequences of Brexit. A Taxing Decision. Economic Policy paper no. 16 April 2016. IMF (2016) United Kingdom. Selected Issues. IMF Country Report no.16/169

2 See Lawless and Morganroth, ESRI, 2016. This ESRI working paper examines the trade of EU countries at a high level of detail and calculates the potential loss of trade due to a combination of WTO tariffs and the estimated elasticity of demand for each product.

3 In a period like 2000-2013 the impact of EU membership on those countries which were members in every year of the period (in this case the EU15 countries) would be included within the pairwise country fixed effects coefficients α_{ij}.

4 The Treasury has not been willing to provide full details of their equation despite several requests.

5 H M Treasury April 2016 table A1.5 page 161. Zero vales for trade are usually dropped in a trade equation in which the dependent variable is logged since the log of zero is indeterminate.

6 Coefficient elasticities are converted to percent changes by the formula $100\left(\frac{X_1-X_0}{X_0}\right)$ where $X_1 = \exp\big(0.766(D=1)\big) = 2.15$, $X_0 = \exp\big(0.766(D=0)\big) = 1$ and D is the value of the binary variable.

7 To allow for additional caution in the lower end of the range [of estimates of the impact of EU membership], the H M Treasury analysis assumes that *"only half of the trade effect of going from the EEA to the negotiated bilateral agreement or the WTO rules comes through within 15 years. This effectively implies a slower degree of regulatory divergence. It is cautious because even in these cases some trade barriers, such as the loss of the financial services passport or the effect of new tariffs, would come through quickly. In the EEA case, the full impact of reintroducing a customs border would be felt immediately and so no reduction in the full estimated trade impact is assumed".* H M Treasury April 2016 para A65.

8 See Treasury (April 2016) para 3.47

9 The only Treasury reference to an offsetting impact of a depreciation is in footnote 30 on page 98.

10 OECD (2016) op. cit. Box 4. Trade openness is measured as the ratio of total exports and imports to GDP.

11 Details of the Poisson estimator are given in the section on Estimates without Fixed Effects.

12 IMF (2016) United Kingdom. Selected Issues. IMF Country Report no.16/169.

13 Head and Mayer review other approaches that provide a theoretical foundation for the gravity equation, but we focus on Anderson and van Wincoop because it is commonly cited.

14 We adopt the same notation for comparison with Anderson and van Wincoop (2003). A detailed description of the derivation of equation 4 and definitions of variables are given in Annex A.

15 The Treasury Report does not include a bibliography, so there are discrepancies over the publication dates of these papers.

16 These are country and time fixed effects. One slight difference is that the Treasury equations are fitted from 1948-2013, whereas ours are from 1950-2015. This should have little impact on the results. We also add equations without country fixed effects for comparison. Our equation includes terms for per capita GDP in place of the Treasury's population. We also include a term for membership of FTAs other than the EU.

17 Zero values are left in the equations using Poisson estimators.

18 The time fixed effect refers to all trade and may include the impact on all trade of a diminishing level of WTO tariffs. Since the time dummy also includes average inflation in export prices and the impact of global recessions it is not possible to say how well any tariff effect is identified.

19 Because there are fewer observations in each successive time period, the statistical precision with which the EU effect is measured declines over time. However, each of the values on chart 3 is statistically significantly different both from zero and also from the value estimated for the 1950-2015 period.

20 Glick R (2016) op cit.

21 See footnote 6 for the mapping of estimated coefficients to percent uplift shown in chart 1.

22 Adding a separate dummy variable into this equation for trade-pairs where both members were within the Eurozone generated a negative coefficient on this dummy which was close to significant at the 5% level for periods beginning in either 1980 or 1990 and ending in 20015.

23 IMF United Kingdom: Selected Issues June 1st.2016. Box 1 p39.

24 In technical terms the 68% estimate was derived from an OLS gravity equation with time fixed effects estimated over the period 1995-2015. This estimate was statistically significant. The 8% estimate was derived from a Poisson pseudo maximum likelihood, (PPML) analysis with multilateral resistance terms fitted over the period 2004-14 and with the home GDP term constrained to equal unity. This EU effect was not statistically significant

25 See IMF (June 2016) p.18. and Gudgin et al (2016) .

26 The Poisson quasi-maximum likelihood (or alternatively pseudo Poisson maximum likelihood (PPML)) estimator is a member of the class of generalised linear models in the exponential family, used with count data. In its application to the gravity equation, it allows for zero and very small observations of bilateral trade observations that would otherwise have to be dropped from the dataset. It enables the linear set of independent variables to be related non-linearly to the dependent variable by means of a link function and implies that the variance of the response is related to the mean. The link function for the Poisson is the log function. In a dataset where there are both very large trade values and zero values, the variance of the random error is likely to be proportional to the mean predictor. In the Poisson case the variance is equal to the mean. If the variance is greater than the mean this is described as over-dispersion. A quasi-likelihood function can be formulated incorporating over-dispersion such that the quasi-maximum likelihood properties of the estimator are identical to the maximum likelihood procedure.

27 This contrast in results is the same irrespective of whether the database includes the full set of countries or just the top 60 UK export markets.

28 The figure of 35% is obtained as 100*(exp(0.697-0.404)-1) from the final column of table 4.

29 The estimate of a 24% gain in services is an average across all EU countries. It may also be the case that the effect on the UK may differ from the average, but we have not investigated this.

30 Feyrer (2009) conducts an interesting analysis of the impact of the closure of the Suez Canal in 1968-76 on trade and GDP. This attempts to isolate the impact of productivity changes associated with changes in grade due lengthening sea voyages during the canal closures. The results appear to be dominated by the experiences of India, Pakistan, Sri Lanka and Kenya.

31 Details of the model can be obtained at:
www.cbr.cam.ac.uk/publications/working-papers/2015

32 It is strange that the Treasury did not investigate the impact of the EU on UK trade alone in its report of 2016, because an earlier Treasury paper of 2005, in the public domain, does make such an estimate. Its estimate of the EU average gain, based on a gravity model with sample, 1960-2004, is an uplift of 38% and a gain for the UK of 7%. The sample size in this earlier paper is very small compared with the 2016 study, but it is interesting to note that the Treasury in 2005 thought it reasonable to look for a specific impact on UK trade, whereas in 2016, no attempt was made.

https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/220968/foi_eumembership_trade.pdf

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Annex A Steps in the theoretical derivation of gravity models

(a) Consumer maximization and general equilibrium

Anderson and van Wincoop (2003) begin with a consumer maximisation problem subject to a budget constraint following traditional static consumer theory. Each country produces a single good (*n* goods), fixed in supply and has a supply price. Preferences are linear and homothetic, meaning that the utility function is homogeneous of degree one. Homothetic preferences imply that the optimal ratio of goods depends only on relative prices and not on income or scale. The consumer problem is:

$$U = \left(\sum_i \beta_i^{\frac{1-\sigma}{\sigma}} c_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \lambda\left(\sum_i p_{ij}c_{ij} - y_j\right), i = 1, n \quad (1)$$

For every country *j*, the consumers in that country are optimising their choice across the *i* goods available in fixed supply. FOC conditions are:

$$\frac{\partial U}{\partial c_{ij}} = \frac{\sigma}{\sigma-1}\left(\sum_i \beta_i^{\frac{1-\sigma}{\sigma}} c_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma}\beta_i^{\frac{1-\sigma}{\sigma}} c_i^{-\frac{1}{\sigma}} - \lambda p_{ij} = 0, i = 1, n \quad (2)$$

For any pair of goods:

$$\frac{\frac{\partial U}{\partial c_{1j}}}{\frac{\partial U}{\partial c_{2j}}} = \frac{p_{1j}}{p_{2j}} \quad (3)$$

Or simplifying:

$$\left(\frac{c_{2j}}{c_{1j}}\right)^{\frac{1}{\sigma}} = \left(\frac{\beta_2}{\beta_1}\right)^{\frac{1-\sigma}{\sigma}} \frac{p_{1j}}{p_{2j}}$$

Solving for c_{2j}:

$$c_{2j} = \left(\frac{\beta_2}{\beta_1}\right)^{1-\sigma} \left(\frac{p_{1j}}{p_{2j}}\right)^{\sigma} c_{1j}$$

The solution is identical for every *i*:

$$c_{ij} = \left(\frac{\beta_i}{\beta_1}\right)^{1-\sigma} \left(\frac{p_{1j}}{p_{ij}}\right)^{\sigma} c_{1j}$$

Re-expressing as:

$$p_{ij}c_{ij} = \left(\frac{\beta_i}{\beta_1}\right)^{1-\sigma} \left(\frac{p_{1j}}{p_{ij}}\right)^{\sigma-1} p_{1j}c_{1j} \quad (4)$$

Adding across all *i* goods:

$$\sum_i p_{ij}c_{ij} = \sum_i \left(\beta_i p_{ij}\right)^{1-\sigma} \left(\beta_1 p_{1j}\right)^{\sigma-1} p_{1j}c_{1j}$$

This expression is identical for any country i . Define expenditure in nominal terms as $x_{ij=}p_{ij}c_{ij}$, then:

$$\sum_i \left(\beta_i p_{ij}\right)^{1-\sigma} \left(\beta_i p_{ij}\right)^{\sigma-1} x_{ij=}y_j$$

Solving for x_{ij} gives:

$$x_{ij} = \frac{\left(\beta_i p_{ij}\right)^{1-\sigma}}{\sum_i \left(\beta_i p_{ij}\right)^{1-\sigma}} y_j$$

This can be re-expressed as:

$$x_{ij} = \left(\frac{\beta_i p_i t_{ij}}{P_j}\right)^{1-\sigma} y_j \quad (4)$$

and

$$P_j = \left(\sum_i \left(\beta_i p_i t_{ij}\right)^{1-\sigma}\right)^{-\frac{1}{1-\sigma}} \quad (5)$$

A&W use a market clearing condition to assert that $y_i = \sum_j x_{ij}$. This implies using (4) above:

$$y_i = \sum_j \left(\frac{\beta_i p_i t_{ij}}{P_j}\right)^{1-\sigma} y_j$$

$$= (\beta_i p_i)^{1-\sigma} \sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} y_j$$

Substituting out the $\beta_i p_i$ term above in (4) we obtain:

$$x_{ij} = \frac{y_i}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} y_j} \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} y_j$$

Let $\theta_j = {}^{y_j}/_{y^W}$. Then:

$$x_{ij} = \frac{y_i y_j}{y^W} \frac{1}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} \theta_j} \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma}$$

$$= \frac{y_i y_j}{y^W} \left(\frac{t_{ij}}{\Pi_i P_j}\right)^{1-\sigma} \quad (6)$$

where:

$$\Pi_i = \left(\sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} \theta_j\right)^{\frac{1}{1-\sigma}}$$

Now substitute out the the $\beta_i p_i$ term in (5) above:

$$P_j^{1-\sigma} = \sum_i \left[\frac{y_i}{\sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} y_j}\right] \left(t_{ij}\right)^{1-\sigma}$$

$$P_j^{1-\sigma} = \sum_i \left[\frac{\theta_i y^W \left(t_{ij}\right)^{1-\sigma}}{y^W \sum_j \left(\frac{t_{ij}}{P_j}\right)^{1-\sigma} \theta_j}\right]$$

Hence:

$$P_j^{1-\sigma} = \sum_i \left(\frac{t_{ij}}{\Pi_i}\right)^{1-\sigma} \theta_i$$

which gives:

$$P_j = \left(\sum_i \left(\frac{t_{ij}}{\Pi_i}\right)^{1-\sigma} \theta_i\right)^{\frac{1}{1-\sigma}} \quad (7)$$

which is equation (11) of A&W (2003). At this stage the $\beta_i p_i$ in the numerator and denominator of (4) have been substituted out and the expression in equation (7) is a function of the θ_i and t_{ij} only. The final step in their argument is that if one assumes that trade barriers are symmetric, i.e. $t_{ij} = t_{ji}$, this implies that summing across *i* gives the same result as summing across *j*, which implies that $\Pi_i = P_i$. Using this result in (6) gives us the gravity equation (equation (13) in A&W (2003)):

$$x_{ij} = \frac{y_i y_j}{y^W}\left(\frac{t_{ij}}{P_i P_j}\right)^{1-\sigma} \qquad (8)$$

with the price indices defined implicitly as:

$$P_j^{1-\sigma} = \sum_i P_i^{\sigma-1}\,\theta_i {t_{ij}}^{1-\sigma} \qquad (9)$$

For a total of n countries, the set of equations in (8) comprise n non-linear equations in the $P_j, j = 1, n$ as functions of the θ_i and the t_{ij}, which can be solved by numerical methods.
In matrix notation:

$$\boldsymbol{P^{1-\sigma}} = \boldsymbol{T'I\theta P^{\sigma-1}}$$

where:

$$\boldsymbol{P^{1-\sigma}} = \begin{pmatrix} P_1^{1-\sigma} \\ P_j^{1-\sigma} \\ P_n^{1-\sigma} \end{pmatrix}, \boldsymbol{T} = \begin{pmatrix} t_{11} & \cdots & t_{1n} \\ \vdots & \ddots & \vdots \\ t_{n1} & \cdots & t_{nn} \end{pmatrix}, \boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}$$

and $\boldsymbol{I}$ is the $n \times n$ unit matrix.

It is important to note that although the P_j are described as price indices, they are actually a summation of output weighted trade barrier terms, t_{ij}. The derivation from conventional consumer maximisation of utility leads to a gravity equation which is multiplicative in outputs of the two countries (or regions) and in trade barrier terms. The additional factor compared with the empirical gravity equation is the presence of the multilateral resistance terms.
It is also important to note that since each country’s output of a single good is fixed in supply, the supply prices are assumed to adjust to clear all markets (including the home market). What is not sold to other countries must be purchased domestically. Yet, prices do not figure at all in the estimated gravity model. Further, the derivation implies that output is nominal, not real. Most empirical gravity models use nominal trade but real GDP.

(b) Head and Mayer (2015)

Head and Mayer work from the primitive concept of a gravity equation:

$$X_{ni} = GS_iM_n\phi_{ni}$$

Using their notation, X_{ni} is the trade by exporter i to destination n. S_i is, according to H&M, the “capability” of exporter i as a supplier to all markets, M_n is the characteristics of “destination market n that promotes imports from all sources.” $\phi_{ni}, 0 \leq \phi_{ni} \leq 1$, represents bilateral trade costs and G is a constant which may conveniently be normalised. A structural gravity equation is defined as:

$$X_{ni} = \frac{Y_i}{\Omega_i}\frac{X_n}{\Phi_n}\phi_{ni}$$

where $Y_i = \sum_n X_{ni}$is the value of production and $X_n = \sum_i X_{ni}$ is the value of the importer’s expenditure on imports from all source countries. The multilateral resistance terms are defined as:

$$\Phi_n = \sum_j \frac{\phi_{nj}Y_j}{\Omega_j}$$

$$\Omega_i = \sum_j \frac{\phi_{ji}X_j}{\Phi_j}$$

Roughly speaking, Φ_n is an output weighted sum of relative resistance terms and Ω_i is an expenditure weighted sum.

The form of the structural gravity equation implies that it is related to the primitive equation by the relations:

$$S_i = \frac{Y_i}{\Omega_i} \text{ and } M_n = \frac{X_n}{\Phi_n}$$

H &M say that a structural gravity model is subject to two conditions: the first is that a country's total expenditures must be allocated across all countries exporting to it; the second is that a country's total exports (including to itself) must equal its total production. The first is expressed in terms of import shares such that:

$$X_{ni} = \pi_{ni} X_n$$

where $\pi_{ni} \geq 0$ and $\sum_i \pi_{ni} = 1$. They require that:

$$\pi_{ni} = \frac{S_i \phi_{ni}}{\Phi_n}$$

where $\Phi_n = \sum_j S_j \, \phi_{nj}$. The second condition is that $Y_i = \sum_n X_{ni}$, which is:

$$Y_i = S_i \sum_n \frac{\phi_{ni} X_n}{\Phi_n}$$

Solving for S_i gives:

$$S_i = \frac{Y_i}{\Omega_i}$$

where:

$$\Omega_i = \sum_n \frac{\phi_{ni} X_n}{\Phi_n}$$

Recall that $\Phi_n = \sum_j S_j \, \phi_{nj}$ and substitute the expression for S_i above:

$$\Phi_n = \sum_j \frac{\phi_{nj}}{\Omega_j} Y_j$$

The final step is to substitute back into the bilateral trade share above:

$$X_{ni} = \pi_{ni} X_n = \frac{S_i \phi_{ni}}{\Phi_n} X_n$$

$$X_{ni} = \frac{Y_i}{\Omega_i} \frac{X_n}{\Phi_n} \phi_{ni} = S_i M_n \phi_{ni}$$

This structural gravity model is based on a general equilibrium in which all production in each country is purchased by itself and other countries (exports) and each country imports goods from other countries. The form they choose ensures that they have a 'multilateral trade resistance' term (MTR) in the equation. Like A&W, this specification has the property that relative trade resistance is what influences bilateral trade.

(c) Comparative statics

The key results, derived as equations (8) and (9) above are:

$$x_{ij} = \frac{y_i y_j}{y^W}\left(\frac{t_{ij}}{P_i P_j}\right)^{1-\sigma} \quad (\mathrm{C.1})$$

:

$$P_j^{1-\sigma} = \sum_i P_i^{\sigma-1}\,\theta_i {t_{ij}}^{1-\sigma} \quad (\mathrm{C.2})$$

A&W totally differentiate equation (9) with respect to P_i, θ_i and t_{ij}. The total differential is evaluated at base period values where $t_{ij} = 1$ and therefore where $P_i = 1$. Further, they assume that there is no change in trade barriers within country *i* so that $dt_{ii} = 0$.

Totally differentiate (9):

$$(1-\sigma)P_j^{-\sigma}dP_j = \sum_i \big[(\sigma-1)P_i^{\sigma-2}\theta_i {t_{ij}}^{1-\sigma}dP_i + P_i^{\sigma-1}{t_{ij}}^{1-\sigma}d\theta_i + (1-\sigma)P_i^{\sigma-1}\theta_i dt_{ij}\big]$$

Evaluate at base period:

$$(1-\sigma)dP_j = \sum_i -(1-\sigma)\theta_i dP_i + d\theta_i + (1-\sigma)\theta_i dt_{ij}$$

Or

$$dP_j = \sum_i \theta_i dt_{ij} - \sum_i \theta_i dP_i + \frac{1}{1-\sigma}\sum_i d\theta_i \quad (\mathrm{C.3})$$

The third term on the r.h.s of the above equation sums to zero because the sum of $\theta_i = 1$. Now multiply the equation by θ_j and sum across all *j*.

$$\sum_j \theta_j\,dP_j = \sum_i\sum_j \theta_i\theta_j dt_{ij} - \sum_i \theta_i dP_i \sum_j \theta_j$$

$$= \sum_i\sum_j \theta_i\theta_j dt_{ij} - \sum_i \theta_i dP_i$$

using $\sum_j \theta_j = 1$. Next, use the comparative static that $dt_{ij} = t$ for $i \neq j$ and $dt_{ii} = 0$. The double sum in the first r.h.s. term in the above equation is equal to the sum of all the off-diagonal elements, which equals 1, minus the sum of squares of the leading diagonal:

$$\sum_j \theta_j \, dP_{j=} \left(1 - \sum_i \theta_i^2\right) dt - \sum_i \theta_i dP_i$$

Or

$$2 \sum_j \theta_j \, dP_j = \left(1 - \sum_i \theta_i^2\right) dt$$

Substitute the above result in (A.3):

$$dP_i = (1 - \theta_i) dt - \sum_i \theta_i dP_i$$

$$= (1 - \theta_i) dt - \frac{1}{2}\left(1 - \sum_i \theta_i^2\right) dt$$

$$dP_i = \left(\frac{1}{2} - \theta_i + \frac{1}{2}\sum_i \theta_i^2\right) dt \quad \text{(C.4)}$$

which is equation (14) of A&W. Total differentiation of (A.1) using the same comparative static assumptions give:

$$d\left[x_{ij} \frac{y^W}{y_i y_j}\right] = (\sigma - 1)\big(dt - dP_i - dP_j\big) \quad \text{(B.5)}$$

On substitution for dP_i and dP_j this gives:

$$d\left[x_{ij} \frac{y^W}{y_i y_j}\right] = -(\sigma - 1)\left[1 + \theta_i + \theta_j - \frac{1}{2}\sum_i \theta_i^2 - 1\right] dt$$

Or

$$d\left[x_{ij} \frac{y^W}{y_i y_j}\right] = -(\sigma - 1)\left[\theta_i + \theta_j - \frac{1}{2}\sum_i \theta_i^2\right] dt \quad \text{(C.6)}$$

which is equation (15) of A&W. It follows straightforwardly that equation (16) of A&W is:

$$d\left[x_{ii}\frac{y^W}{y_i y_j}\right] = (\sigma - 1)\left[1 - 2\theta_i + \sum_i \theta_i^2\right] dt \quad (\text{C}.7)$$

since the term in brackets on the r.h.s. of (B.5) for $dt = dt_{ii} = 0$ and $dP_i = dP_j$ Equations (B.4) and (B.6) to (B.7) are the comparative static forms that are the basis for their three implications on page 175 of the trade impacts for trade with large and small countries.

Anderson and van Wincoop stress the theoretical mechanism underlying their interpretation of empirical gravity equations:

> "The economics behind the formal result is that the constant vector of real products must be distributed despite higher trade costs. The rise in trade costs is offset by the fall in supply …. required to achieve shipment of the same volume."

Annex B Gravity Model Equations

In the following equations the variables are defined as follows:

- TRADE Exports between pairs of countries in current $
- GDP_HOME GDP of the exporting country in 1990 $ at PPP.
- GDP_DEST GDP of the importing country in 1990 $ at PPP.
- POP_HOME Population of the exporting country
- POP_DEST Population of the importing country
- DIST Distance between the two countries in each trade-pair
- MEANDIST_HOME Mean distance between the exporting country and all trade partners
- MEANDIST_DEST Mean distance between the importing country and all of its trade partners
- EU_BOTH Dummy variable for both countries in a trade-pair being members of the EU
- EU_HOME Dummy variable for exporting country in a trade-pair being an EU member
- EU_DEST Dummy variable for importing country in a trade-pair being an EU member
- UK_EU Dummy variable for UK exports to an EU member count
- FTA_BOTH Dummy variable for both countries in a trade-pair being members of a non-EU FTA
- NOR_EU Dummy variable for Norway exports to an EU member country
- SWZ_EU Dummy variable for Swiss exports to an EU member country
- CHINESE Dummy variable for both countries in a trade-pair speaking Chinese
- GERMAN Dummy variable for both countries in a trade-pair speaking German
- PORTUGESE Dummy variable for both countries in a trade-pair speaking Portuguese

Equation B.1 all countries with fixed effects

Dependent Variable: LOG(TRADE)
Method: Panel Least Squares
Date: 04/22/17 Time: 16:41
Sample: 1950 2015
Periods included: 66
Cross-sections included: 13934
Total panel (unbalanced) observations: 459211
White cross-section standard errors & covariance (d.f. corrected)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-4.823768	0.756781	-6.374061	0
LOG(GDP_HOME)	0.523355	0.058644	8.924317	0
LOG(GDP_DEST)	0.949033	0.03969	23.91125	0
LOG(GDP_HOME/POP_HOME)	1.06506	0.067149	15.86124	0
LOG(GDP_DEST/POP_DEST)	0.173526	0.04522	3.837354	0.0001
UK_EU	-0.237284	0.040052	-5.924374	0
EU_BOTH	0.779366	0.046543	16.74523	0
EU_HOME	0.151847	0.025798	5.886016	0
EU_DEST	-0.136111	0.030734	-4.428642	0
FTA_BOTH	0.389617	0.083916	4.642923	0
NOR_EU	0.448122	0.044191	10.1405	0
SWZ_EU	0.754907	0.049056	15.38882	0

Effects Specification

Cross-section fixed (dummy variables)
Period fixed (dummy variables)

R-squared	0.841919	Mean dependent var	15.76788
Adjusted R-squared	0.836945	S.D. dependent var	3.44795
S.E. of regression	1.392286	Akaike info criterion	3.529805
Sum squared resid	863004.2	Schwarz criterion	3.866539
Log likelihood	-796452.5	Hannan-Quinn criter.	3.625469
F-statistic	169.2543	Durbin-Watson stat	0.664574
Prob(F-statistic)	0		

Equation B.2 Top 60 UK export markets with fixed effects

Dependent Variable: LOG(TRADE)
Method: Panel Least Squares
Date: 04/23/17 Time: 11:52
Sample: 1950 2015
Periods included: 66
Cross-sections included: 3639
Total panel (unbalanced) observations: 165860

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.993771	0.258090	-23.22354	0.0000
LOG(GDP_HOME)	0.699189	0.017935	38.98522	0.0000
LOGAGDP_DEST)	0.992415	0.015626	63.51205	0.0000
LOG(GDP_HOME/POP_HOME)	0.889323	0.019493	45.62179	0.0000
LOG(GDP_DEST/POP_DEST)	-0.057167	0.015801	-3.617883	0.0003
UK_EU	-0.373943	0.069174	-5.405832	0.0000
EU_BOTH	0.615816	0.020943	29.40435	0.0000
EU_HOME	0.152631	0.013241	11.52689	0.0000
EU_DEST	-0.088981	0.013964	-6.372032	0.0000
FTA_BOTH	0.586295	0.048752	12.02597	0.0000
NOR_EU	0.168964	0.050200	3.365792	0.0008
SWZ _EU	0.499384	0.050226	9.942774	0.0000

Effects Specification

Cross-section fixed (dummy variables)
Period fixed (dummy variables)

R-squared	0.883852	Mean dependent var	17.63886
Adjusted R-squared	0.881192	S.D. dependent var	3.075011
S.E. of regression	1.059913	Akaike info criterion	2.976395
Sum squared resid	182156.4	Schwarz criterion	3.200803
Log likelihood	-243117.5	Hannan-Quinn criter.	3.042985
F-statistic	332.2227	Durbin-Watson stat	0.516571
Prob(F-statistic)	0.000000		

Equation B.3 all countries without fixed effects

Dependent Variable: TRADE
Method: Generalized Linear Model (Newton-Raphson / Marquardt steps)

Date: 04/23/17 Time: 20:41

Sample: 1950 2015

Included observations: 722366

Family: Poisson Quasi-likelihood

Link: Log

Dispersion fixed at 1

Convergence achieved after 7 iterations
Coefficient covariance computed using the Huber-White method with observed Hessian

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-12.96323	0.131690	-98.43779	0.0000
LOG(GDP_HOME)	0.851496	0.005066	168.0877	0.0000
LOG(GDP_DEST)	0.862108	0.005124	168.2519	0.0000
LOG(GDP_HOME/POP_HOME)	0.449555	0.010018	44.87443	0.0000
LOG(GDP_DEST/POP_DEST)	0.447144	0.007786	57.43147	0.0000
LOG(DIST/(MEANDIST_HOME*MEANDIST_DEST))	-0.819123	0.008534	-95.98123	0.0000
CONTIG	0.109109	0.018964	5.753385	0.0000
UK_EU	-0.530022	0.028728	-18.44986	0.0000
EU_BOTH	0.756215	0.014136	53.49502	0.0000
FTA_BOTH	0.217137	0.039998	5.428669	0.0000
NOR_EU	0.505767	0.043613	11.59667	0.0000
SWZ_EU	0.317785	0.031014	10.24659	0.0000
CHINESE	2.580000	0.060013	42.99053	0.0000
GERMAN	0.496659	0.031686	15.67432	0.0000

Mean dependent var	3.55E+08	S.D. dependent var	4.06E+09
Sum squared resid	2.66E+24	Quasi-log likelihood	5.60E+15
Restr. quasi-logl	4.79E+15	Quasi-LR statistic	1.61E+15
Prob(Quasi-LR stat)	0.000000	Pearson SSR	3.47E+14
Pearson statistic	4.80E+08	Dispersion	1.000000

EU_HOME and EU_DEST terms are omitted from this equation since they were statistically insignificant for all time periods.

Equation B.4 Top 60 UK export markets without fixed effects

Dependent Variable: TRADE
Method: Generalized Linear Model (Newton-Raphson / Marquardt steps)
Date: 04/23/17 Time: 21:39
Sample: 1950 2015
Included observations: 222565
Family: Poisson Quasi-likelihood
Link: Log
Dispersion fixed at 1
Convergence achieved after 6 iterations
Coefficient covariance computed using the Huber-White method with observed Hessian

Variable	Coefficient	Std. Error	z-Statistic
C	-12.27506	0.159598	-76.91218
LOG(GDP_HOME)	0.826338	0.0059	140.051
LOG(GDP_DEST)	0.853131	0.006386	133.601
LOG(GDP_HOME/POP_HOME)	0.46809	0.011788	39.70844
LOG(GDP_DEST/POP_DEST)	0.494855	0.009537	51.88939
LOG(DIST/(MEANDIST_HOME*MEANDIST_DEST))	-0.777259	0.010001	-77.72023
CONTIG	0.122036	0.021415	5.698675
UK_EU	-0.606357	0.035313	-17.17096
EU_BOTH	0.688427	0.016305	42.22161
EU_HOME	-0.065289	0.015008	-4.350408
FTA_BOTH	0.24813	0.04203	5.903704
NOR_EU	0.481903	0.04465	10.79296
SWZ_EU	0.284861	0.032039	8.890971
CHINESE	2.615578	0.0623	41.98331
PORTUGESE	0.535359	0.117611	4.551949
GERMAN	0.577816	0.03253	17.76241

Mean dependent var	1.04E+09	S.D. dependent var	7.23E+09
Sum squared resid	2.59E+24	Quasi-log likelihood	5.12E+15
Restr. quasi-logl	4.57E+15	Quasi-LR statistic	1.09E+15
Prob(Quasi-LR stat)	0	Pearson SSR	2.37E+14
Pearson statistic	1.06E+09	Dispersion	1