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Abstract

A language competition situation between a dominant majority language and a minority language is modeled, where the minority language group is a result of a continuous and ongoing migration process. Migrants enter the society, form families and produce offspring. Children are raised in either one of the two languages or both, visit school, learn additional languages as adults and produce a next cohort with its own linguistic repertoire. Families and adults are conceptualized as utility maximizing actors, who take into account instrumental aspects of languages, as their communicative range, as well as identity related aspects.

A general macro-level model is proposed that describes how the linguistic composition of a population facing migration changes over time. Furthermore, a specific functional form of the general model is proposed and steady states are analyzed. Finally, some numerical examples are presented.

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1 Introduction

Language competition models are formal mathematical models that describe how the (relative) size of certain language groups or the geographical distribution of certain languages changes over time within a given territory. In the past two decades a growing number of such models were developed. They borrow techniques and ideas from biology, chemistry, physics and economics, and use respective metaphors to describe and simulate the (aggregated) linguistic behavior of human beings.

Many language competition models are concerned with the decline and death of languages historically rooted in specific regions. This is motivated by the observation that "[a]ll over the world, people are stopping speaking minority languages and shifting to languages of wider communication" (Sallabank, 2012, p. 104). In contrast, we here focus on minority languages (from the host country point of view) mostly spoken by “newcomers” or their descendants.

We confine ourselves to states or regions with one official or dominant language facing an ongoing inflow of speakers of another language. Throughout the paper we call the locally dominant majority language H and the minority language of the newcomers L. The model developed in this paper describes how the number of speakers of H and L and, hence, the linguistic composition of the population change over time. External mobility, i.e. the inflow of new individuals with certain language repertoires, is one mechanisms driving the language dynamics explicitly modeled here. A second mechanism that affects the evolution of the linguistic composition is intergenerational language transmission. Individuals form couples and families, have children and transmit one or more of the languages they speak to their children. For these processes *emotional* and *instrumental* motives as well as external conditions such as linguistic concentration/segregation play an important role and are included in the model. A third mechanism taken into account is language learning. On the one hand, young newcomers or children of newcomers enjoy formal education in one of the two languages or both – in practice predominantly in H – and might become bilingual or even monolingual in H. Adult newcomers might acquire the locally dominant language over the years to improve their social and economic opportunities. On the other hand, monolinguals in the dominant language H can also acquire the minority language L. The model proposed here comprises these three mechanism to describe the dynamics of a population that is constantly changing.

The paper is organized as follows. After a short literature overview, central variables are introduced, the linguistic environment is operationalized and the effects of the environment on language transmission and acquisition are discussed. In the following sections the general model formulation is presented and specified. Then, steady states of the model are analyzed. In the last section, some numerical examples shall illustrate the language dynamics graphically and show how a change in model parameters affects medium- and long-term outcomes. The paper finishes with some conclusions.

2 Language Competition Models: A Short Overview

In the literature on language dynamics models we find two main strands of research. The first one is influenced by economic theory. Economic approaches are characterized by models of individual agency: agents aim at maximizing their individual utility when making language related decisions. One of the first dynamic language economics models was presented in Grin (1992). There, individual agents can conduct activities in one of two languages and gain utility from them. Activities as well as language learning require the investment of time and goods, while both are limited. The linguistic decisions of individual agents then drive the overall language dynamics. A second model to be mentioned here was developed in John & Yi (1996). There, a two period model for two monolingual regions with

different languages is developed. In every period individuals can either learn the other language or engage in production. Between the periods, they are able to move to the other region and engage in production there. A dynamic version of the two-period model with successive generations can be found in John & Yi (2001). In Tamura (2001) the agents are families: parents decide on spending time to raise their children as bilinguals, which might have a positive impact on the child's income, or to spend their time on consumption. Kennedy & King (2005) construct a three generation model with overlapping generations, where adults vote for public language policies. As before, for production adults profit from others speaking their language(s) and are hence interested in educational policies that teach their language to children. At the same time education is costly, such that adults have to weight costs against benefits. A last set of models to be mentioned here applies a game theoretic approach. Although not all of them are actual dynamic models, they inspired many of the previously listed economic models. In 1991 Selten and Pool published a game-theoretical model that can account for an arbitrary number of languages. Here, individuals gain communicative benefits from learning new languages. Adoptions, extensions and applications of the Selten-Pool model or comparable game-theoretical approaches can, for example, be found in Church & King (1993), Gabszewicz *et al.* (2011) and Iriberri & Uriarte (2012).

A second strand of research startet with a short paper published in 2003: "It was the population dynamics model of Abrams and Strogatz [...] which started the avalanche of physics papers on language competition" (Schulze *et al.*, 2008, p. 13). In their "minimal model" two languages compete for monolingual speakers. The attractiveness of a language is determined by the number of its speakers as well as its "perceived status". A simple differential equation is provided that describes the language dynamics. On the one hand, Abrams and Strogatz could fit their "minimal model" to aggregated empirical data of endangered languages. On the other hand, the model does not really account for the underlying social and linguistic processes of language acquisition, transmission and change, neglects bilinguals and predicts that always one of the two competing languages will become extinct in the long run. Due to such shortcomings the Abrams-Strogatz (AS) model was revised and extended by many authors, especially from the field of (statistical) physics. Patriarca & Leppänen (2004), Patriarca & Heinsalu (2009) and Patriarca *et al.* (2012) include spatial components in their adaptations of the AS model. Mira & Paredes (2005) introduced the idea of similarity between competing languages to the original model and Stauffer *et al.* (2007) propose individual based versions of the AS model and apply simulation techniques instead of averaging over the whole population. Furthermore, Mira & Paredes (2005), Wickström (2005), Minett & Wang (2008), Heinsalu *et al.* (2014) and others extended the A-S model by additionally considering bilinguals. The role of bilinguals in language shift is crucial, since in practice individuals normally do not change from being monolingual in one language to being monolingual in the other language. Instead, it is far more realistic to consider transitions from monolinguals in one language to bilinguals and - in a second step - from bilinguals to monolinguals in the other language. For such a two step transition it normally takes more than one generation. For that reason, some authors explicitly modeled vertical, i.e. intergenerational, language transmission. Minett & Wang (2008) develop a uni-parental model for vertical language transmission. In contrast, Wickström (2005), Fernando *et al.* (2010), Templin *et al.* (2016) and others use a two-parents model. Here adult individuals (randomly) form families and bring up their children in either one of the parents languages or in both of the parents languages. An other feature of the model developed in Minett & Wang (2008) is horizontal language transmission. During their lifetime monolingual adults can learn a second language and therefore become bilingual. In their spatial model, Patriarca *et al.* (2012) also consider bilinguals. In their simulations small bilingual areas emerge at the boundary between larger monolingual areas of both languages.

In contrast to Abrams and Strogatz, Pinasco & Romanelli (2006) proposed a Lotka-Volterra type model[1] inspired by population dynamics to model language competition. Spatial extensions of this model can be found in Kandler & Steele (2008) and Kandler *et al.* (2010). Zhang & Gong (2013) also use a Lotka-Volterra type model and offer an alternative to abstract status parameters.

For more extensive literature reviews, see Patriarca *et al.* (2012), Gong *et al.* (2014) and John (2016).

In the present paper we propose a model that combines aspects of both strands of research. We consider monolinguals of two languages as well as bilinguals. Adult individuals form families (two-parent model) and transmit one or both languages to their children. Furthermore, two different modes of horizontal language transmission are taken into account. As in Minett & Wang (2008), adult individuals can learn new languages to improve their human capital. New is that we explicitly model language acquisition in formal education. To obtain mathematical models for the different processes of language transmission and acquisition we do not use physical or biological analogies. Instead, families and adult individuals are conceptualized as utility maximizing agents. Take parents speaking the minority language. On the one hand, many of them want to transmit their heritage language to the next generation. On the other hand, parents want their children to have good social and economic perspectives, which are often bound to proficiency in the dominant language. This results in a possible trade-off between instrumental and identity related motives. Comparably, adults have to weight benefits (improved human capital) against learning costs. As in Wickström (2005), this utility-maximization approach is used to derive properties of the transmission and acquisition processes. Moreover, we offer a possible operationalization of the most often abstract and vague status parameter used in many models. Another novelty of the present approach is that the effect of external mobility (the inflow of new individuals speaking a "new" minority language) on the language dynamics is explicitly analyzed.

3 Towards a Multidimensional Model

This section is devoted to the main variables, parameters and mechanisms of the language dynamics model proposed later on. We start with 'the number of speakers' as the central object of interest and how it is operationalized. Thereafter, we discuss how the dynamics are conceptualized. Next, processes of language transmission and acquisition are discussed in greater detail.

3.1 Numbers of Speakers and Linguistic Composition

We are interested in the distribution of certain language repertoires[2] throughout a population or a given territory. As the vast majority of models available in the literature, we consider two languages, a locally dominant high-status majority language H and a low-status minority language L. Here, high and low status is meant from a local point of view: L might be the dominant language in another country. Furthermore, three language repertoires are considered: monolingualism in H, monolingualism in L and bilingualism. Individuals are grouped together according to their language repertoires. By X_H we denote the fraction of the population that are H-monolinguals, by X_L the fraction that are L-monolinguals and by X_B the fraction of bilinguals. Note that $X_B = 1 - X_H - X_L$, i.e. that all language repertoire groups together make up the whole population. Hence, the vector $X := (X_H, X_L)$, which is the central dependent variable in our model, fully characterizes the linguistic composition of the population.

At a first glance it might seem straight forward for non-linguists to assign every individual to one of those three language repertoires. But what are the minimal competences one has to have to count as a "speaker" of a language? While many papers do not explicitly discuss this issue, Wickström (2005) and Fernando *et al.* (2010) provide two possible definitions of mono- and bilingualism. To be a bilingual in the sense of Wickström (2005), one has to be raised bilingually. According to (Fernando *et al.*, 2010, p. 53), however, bilingualism is defined as "the ability to function confidently in two languages, that is, the ability to have communicative competence in two languages" (p. 53). Since we emphasize the role of language as a tool for communication and as human capital and since we want to take different modes

of language acquisition into account, a functional understanding of proficiency appears to be the most appropriate here. We thus follow the definition of bilingualism given by Fernando *et al.* (2010).

3.2 Dynamics

Language dynamics models are concerned with the evolution of the linguistic composition of a population over time. Models make (probabilistic) statements about the composition of "tomorrow", given today's composition as well as other relevant information. In formal terms this reads as follows. Let t denote time. We write $X_{LR}(t)$ for the fraction of the population having language repertoire LR, $LR = H$, L or B, at time t. Given $X(t) := (X_H(t), X_L(t))$ and other relevant information, a language dynamics model makes statement about $X(t+1)$.

The linguistic composition of a population evolves due to a number of complex processes. As every model builds on a necessary simplification of the complex reality, we concentrate on four key processes. First, individuals enter or leave the population. We call this population dynamics. On the one hand, children are born and people die. On the other hand, individuals from other states move to the state of consideration, or vice versa. Second, children acquire one or several languages mostly within the family context. Third, pupils are educated in a certain language and can learn additional languages in formal education. Fourth, adult individuals can learn new languages, improve their skills in a language, or can forget a language they do not use regularly.

Neglecting population dynamics for the moment, the evolution of the linguistic composition is conceptualized as an aggregated result of individual language related decisions. These individual decisions are framed by the linguistic environment, which includes the linguistic composition of the population.[3] From an individual point of view, the linguistic environment is exogenous. Nevertheless, individuals actively shape the linguistic environment through their decisions. To put it differently, individual language related behavior is not isolated but affected - at least to some extent - by the behavior of the rest of the population. This is modeled as follows. At each point in time t individuals in a population make language related decisions. These decisions are framed by the linguistic environment at time t. In turn, the linguistic behavior of the population, understood as aggregated individual decisions, may change the linguistic environment at the next point in time $t+1$. This influence of todays behavior on the behavior of tomorrow reflects the network effects.

3.3 Language Transmission and Acquisition

This section is devoted to the three processes of language transmission within the family, language education and adult language learning.

3.3.1 Family Formation and Intergenerational Language Transmission

We restrict ourselves to language acquisition within families with two parents and explicitly model family formation. As in Wickström (2005) and Fernando *et al.* (2010) we do not differentiate between the language repertoire of the mother and the one of the father and hence get six possible family types F ($F = HH, HB, LL, LB, BB, HL$). Family formation is assumed to be the result of a random search and mating process: adult individuals meet randomly, some form couples and some of these couples have children. While Wickström (2005) assumes a low success probability for HL couples, i.e. couples of a monolingual speaker of H and a monolingual speaker of L, Fernando *et al.* (2010) neglect this unlikely family type, i.e. assume success probability 0, due to the immense communicational barriers (if one of

the partners acquires the other language, then the couple is not HL anymore). Here, we follow Fernando *et al.* (2010). For the mathematical details on how family formation is modeled, see Section 4 and the Appendix.

Next, we deal with intergenerational language transmission. In Tamura (2001) parents can use their spare time to teach their children a second language or to do something else. Both using spare time for something else and facilitating a better economic perspective for their children (due to bilingualism) yield utility for the parents. Similarly, in Wickström (2005) parents also choose the language repertoire LR for their children, which yields the highest individual utility. But for Wickström it is not the free time parents are weighting against their children's economic future. Instead, parents want to pass cultural values attached to their heritage language(s) to the next generation. If the language that promises economic opportunities and the heritage language are not identical, then there is a similar trade-off as the one between consumption (of free time) and production (of bilingual children) analyzed in Tamura (2001). In Wickström (2005), the more people speaking a language and the higher the status of that language, the higher is the individual utility one gains from that language.

As in Templin *et al.* (2016), we here adopt the Wickström model for individual language transmission. Counting the number of all F-type families choosing language repertoire LR yields $q_{LR}(F)$, the fraction of F-type families bringing up their children in language repertoire LR. It is assumed that only those languages spoken by the parents can be transmitted. That implies, that HH families can not transmit L to their children. Furthermore, it is assumed that both parents shall be able to speak with their children. Hence, the children has to share at least one language with both parents.

Since a child's peer group affects its attitudes towards and exposure to the minority language, linguistic concentration (often connected to ethnic concentration) in neighborhoods and (pre-)schools can have positive or negative effects on the success of language transmission.

3.3.2 Language Learning in Formal Education

"One truth is certain: formal schooling introduces perhaps the most important outside influence (for better or worse) on a family's strategy to rear multi-literate/multilingual children [...]" (Caldas, 2012, p. 357). The daily language(s) of the classroom, the foreign languages taught and the language repertoires the peers have a strong effect on pupils future language repertoires. As for transmission in the family, parents and their children have to make certain choices with respect to language and education. First, parents can send their children to schools with different linguistic models, e.g. schools dominated by H, schools dominated by L, immersion schools and bilingual schools of different types.[4] Obviously, there is only a choice if different school types are available, which is by far not always the case. Often, education is only available in the majority language, which hampers the maintenance of the minority language. Over the years children are learning abstract ideas and concepts and their vocabulary in the majority language grows. At some point, they can express themselves better in the majority language, since the daily conversations at home might not be able to compete with this enlarged vocabulary, see, e.g., Okita (2002, p. 125). If courses or programs in the minority language are offered, the maintenance is much easier and, additionally, some H-monolingual might acquire L during their school career.

As before, there is a possible trade-off for parents. To transmit the minority language in an environment dominated by the majority language, a L-monolingual school might be the best option. At the same time, the majority language is normally related to better socio-economic opportunities for the children, and it is more likely to be the language of upward mobility. Hence, from an instrumental point of view, an H-monolingual school might appear be the best option.[5] Although decisions are made by parents and children, we would argue that – from a macro perspective – education policies with respect to language

are more important than individual decisions. If there is only one option, for example, then there is no choice. To account for the importance of education policies and not to complicate the model even further, we therefore assume language learning at school to depend only on external (policy) factors, but not on the number of speakers or on the status of the minority language.

3.3.3 Language Learning by Adults

The majority of children growing up in an environment dominated by one (official) language (H) do acquire this language. This is different for adults moving to such an environment (temporarily or permanently) at a certain age but with little or no proficiency in H. Even if they stay for many years, they might never 'fully' acquire the local language.

Chiswick & Miller (2002) developed a useful model in which proficiency in H is a function of exposure to the language, efficiency in learning the language and economic incentives (three E's). The model is based on a human capital approach. Esser (2006) provides a similar model that also builds on a human capital approach. We shortly outline the three E's model as well as the results of the quantitative analysis presented in both papers.[6]

The 'value' of the human capital 'language' or 'language skills' depends on the linguistic environment. With proficiency in H in an H-dominated environment one can find better jobs and be more productive in the job. Furthermore, proficiency enables more efficient consumption and communication with locals and local authorities/administration. This yields certain incentives to learn H. At the same time, language learning is costly, it requires the investment of time and other resources.

Exposure to the local language facilitates its learning in two ways. First, being exposed to and addressed in H on a regular basis yields an additional incentive to learn H, compared with a situation in which a newcomer can use L in her everyday live. Living in an area with many other speakers of L reduces the communication costs for L-monolinguals. If there is a local economy offering jobs in L, the incentives to learn H are even lower. Hence, the linguistic concentration (of L-speakers) is found to have a negative effect on H-proficiency, cf. Chiswick & Miller (2002) and Esser (2006). Furthermore, migrants with little proficiency in H choose to move to linguistic enclaves because of their low proficiency. Secondly, the more one is exposed to a language, the more opportunities there are to use that language and hence the more practice one can get. Besides linguistic concentration, time since migration is identified as another important factor for exposure to H. Concerning the motivation (incentives) to learn H it was found that those newcomers who plan to stay for a long time are more likely to acquire H. Efficiency variables are age at time of migration (with increasing age language learning decreases) and education.

3.4 Variables, Parameters and Formal Relations

Based on the previous deliberations, the aim of this section is to propose a number of relevant variables that characterize the linguistic environment and the language repertoire groups. We specify how these quantifiable variables affect the processes of language transmission and learning at the macro level. This yields the foundation for our mathematical language competition model.

Notation: As before, language repertoires are denoted by LR, $LR = H, L, B$. If the language is not specified, we use l, $l = H, L$.

Recall, by $X = (X_H, X_L)$ we denote the linguistic composition of the population. The fraction of bilingual individuals X_B is given by $1 - X_H - X_L$. The linguistic composition X evolves due to family formation, language transmission within the family, language education and adult language learning. For the mathematical model this translates into four quantities determining the dynamics of X:

- ψ_F: fraction of F-type families,
- $q_{LR}(F)$: fraction of F-type families bringing up their children with language repertoire LR,
- s_{LR_1,LR_2} fraction of children entering school with language repertoire LR_1 and leaving school with language repertoire LR_2,
- a_{LR_1,LR_2} rate at which adults with language repertoire LR_1 become adults with repertoire LR_2.

Note, q_{LR}, s_{LR_1,LR_2} and a_{LR_1,LR_2} can also be interpreted as probabilities: $q_{LR}(F)$ can be seen as the probability that a child growing up in an F-type family develops language repertoire LR. These quantities are not just fixed numbers but depend on the linguistic composition and various aspects of the linguistic environment. We propose two main parameters to characterize the linguistic environment.

3.4.1 Linguistic Environment

As outlined earlier, family formation is modeled as a random search and mating process. On the one hand, the result of this process clearly depends on the linguistic composition of the population. On the other hand, who actually meets whom is not just a question of the overall numbers. Spatial concentration or even segregation of the minority language group might favor monolingual couples. Furthermore, linguistic concentration affects the amount of exposure to both languages and hence intergenerational language transmission, see above. Therefore we introduce a variable $C \in [0, 1]$, a measure for the concentration of speakers of language L, i.e. L-monolinguals as well as bilinguals. Treating bilinguals rather as speakers of L than as speakers of H is justified by the assumption that most L-monolinguals respectively their descendants acquire the dominant language H, while only a small number of H-monolinguals learn the minority language L.[7] Thus, the vast majority of bilinguals is assumed to have L antecedents and hence some feeling of belongingness to the L-community. Furthermore, such a conception of bilinguals reflects the frequently encountered connection between language and ethnicity. For $C = 0$ the language repertoire groups are distributed equally throughout the territory of consideration (no concentration). If $C = 1$, then we are dealing with a spatially segregated population (maximal concentration).

One question that remains is how to define and measure concentration. Segregation – and hence concentration – is a "multidimensional phenomenon varying along five distinct axes of measurement: evenness, exposure, concentration, centralization, and clustering" (Massey & Denton, 1988, p. 281). This is one reason why in the literature a number of different indices for spatial segregation can be found. It is out of the scope of this paper to give a detailed overview on the discussion of how to measure segregation. Instead, we refer the interested reader to the literature, see e.g. Massey & Denton (1988) or Morrill (2016). One possible concentration/segregation measures are the index of dissimilarity, which measures the proportion of the minority population that has to move to achieve an equal distribution of L's, see Morrill (2016).

We have seen that when bringing up children in an H-dominated setting, parents speaking the minority language have to weight objective criteria against emotional motives: there is "a trade-off between the efficiency aspect of language as a means of communication, on the one hand, and its importance as a carrier of identity, on the other hand" (Wickström, 2005, pp. 83). Conceptualized as rational actors, parents choose the language repertoire for their children that yields the highest utility. In Wickström (2005) and Templin *et al.* (2016) the communicational value (efficiency aspect) of a language is captured by the number of speakers of that language. The identity aspect is captured by status parameters. While Wickström (2005) works with two status indicators, Templin *et al.* (2016) use a single status parameter that depicts the relative status of the minority language. Similar status or prestige parameters can be found

in many language competition models. One problem with such parameters is that normally no proper interpretation for their concrete values is provided. To illustrate this fact, Fernando et al. ask "what were the characteristics of a language having a prestige value, say 1.2, and what was the sociocultural condition corresponding to the difference between two languages having prestige values, say 1.2 and 1.3, respectively" (Fernando *et al.*, 2010, p. 50)? A similar criticism is expressed in Zhang & Gong (2013). To get a more meaningful status variable, we first identify different dimensions of the status of a language. For this purpose, the systematic framework first developed in Giles *et al.* (1977) is very helpful. There, the authors suggest three categories of factors that are involved in language vitality: demographic factors, status factors and institutional support factors. Demographic factors are related to the number of speakers as well as their spatial distribution. The main status factors are economic, social and symbolic status. The economic status displays the economic standing of the language groups, the social status is related to prestige, social standing and (political) power and the symbolic status is related to identity and culture, cf. Baker (2011, pp. 55f). Institutional support comprises government (services) on different levels, mass media, business and education. For the present model we concentrate on the socio-economic and the institutional dimension.

First, the socio-economic position of a language repertoire group is measured by the average socio-economic status (SES) of all individuals belonging to that group. The SES is a widely used variable in econometric and sociological analyses and can be measured in different ways. The main dimensions of the SES are income, education and occupation. Let $\overline{S}_{SE}(L) \in [0,1]$ denote the average normalized SES of people speaking L, and $\overline{S}_{SE}(L) \in [0,1]$ the average normalized SES of H-monolinguals. As in Templin *et al.* (2016) we are interested in the socio-economic advantage individuals gain from speaking a certain language. Hence, instead of just asking how high the status of a certain language is, we ask how high is the status of one language compared with the other one. Accordingly, we define the relative the socio-economic status S_{SE}. Let $l \in \{H, L\}$. The relative socio-economic status of language repertoire group l is defined by

$$S_{SE}(l) := \frac{\overline{S}_{SE}(l)}{\overline{S}_{SE}(H) + \overline{S}_{SE}(L)}.$$

Note, $S_{SE}(H)$ reflects the socio-economic status of H-monolinguals, while $S_{SE}(L)$ is the socio-economic status of individuals speaking L.

Second, we consider the *institutional* or *official* status of a language. This status variable is determined by *official* domains the language can be used for. The *official* status is oriented towards the graded intergenerational disruption scale (GIDS), see Fishman (1991). The *official* status clearly belongs to institutional support factors in the Giles *et al.* framework. Above that, the *official* status might reflect the prestige of a language to some extent and, at the same time, affect the prestige of the language. For example, providing governmental services in a minority language can improve its perceived status. This illustrates, how the different categories of variables are strongly connected to one another. For the design of a single variable that captures various aspects of the interaction between individuals with a certain language repertoire and *official* institutions, we first have to provide a list of d domains. For each domain we make the simplifying assumption that a language can either be used or not used. Counting the number of domains a language l, $l = H, L$, can be used for and dividing this number by d, the total number of domains, we obtain a status variable $\overline{S}_{OF}(l)$ with possible values in $\{0, 1/d, 2/d, ..., (d-1)/d, 1\}$. If $S_{of}(l)$ equals 1, then the language can be used in all d domains, which might be the case for the dominant local language. Since education is treated separately, we here consider three domains: government at national level, government at provincial level and public administration. This yields an *official* status variable $\overline{S}_{OF}(l)$, $l = H, L$, with possible values in $\{0, 1/3, 2/3, 1\}$. Since H is the locally dominant and official/national language we assume $\overline{S}_{OF}(H)$ to be one, i.e. H can be used in all domains. As for the

socio-economic status, we introduce the relative official status

$$S_{Of}(l) := \frac{\overline{S}_{OF}(l)}{\overline{S}_{OF}(H) + \overline{S}_{OF}(L)} = \frac{\overline{S}_{OF}(l)}{1 + \overline{S}_{OF}(L)}.$$

Although the socio-economic status and the official status of the minority language are clearly related to *objective criteria*, both status variables are also connected to the prestige of that language. So, are these two status variables sufficient to account fully for the *symbolic* status of the minority language, and the *emotional motives* that play an important role in its intergenerational transmission? Probably not. We have seen that many migrant parents want to pass their first language on to their children despite its economic and official status, since this is the language they are most comfortable in and that is associated with their (cultural) heritage and (group) identity. But how to quantify and measure this cultural dimension? The major problem lies in the difficulty, if not even impossibility, of proposing a single number that adequately describes the average attachment to, or *value* of, the heritage language for its speakers. "The components of non-market value [of a language] are very difficult to identify theoretically, and no less difficult to measure empirically [...]" (Grin, 2003, p. 38). For that reason, we restrict ourselves to the socio-economic status and the official/institutional status for the current model. We leave it to future research to extend the current model with further parameters that display more dimensions of the status of the minority language - as community support for the minority language or presence of both languages in the media - to get a more complex model.

To simplify the formulas in the upcoming section we introduce a single status indicator that is composed of the two status parameters. Let α_{SE} and α_{OF} be weights, i.e. $\alpha_{SE}, \alpha_{OF} \in [0, 1]$ and $\alpha_{SE} + \alpha_{OF} = 1$. Given such weights we define the composed status indicators, $l = H, L$

$$S(l) := \alpha_{SE} \cdot S_{SE}(l) + \alpha_{OF} \cdot S_{OF}(l). \tag{1}$$

Since both single status parameters are normalized, the composed status indicators have values between 0 and 1 as well. Furthermore, we have $\overline{S}_{SE}(H) = 1 - \overline{S}_{SE}(L)$, $\overline{S}_{OF}(H) = 1 - \overline{S}_{OF}(L)$. Therefore, we get $S(L) = 1 - S(H)$. The composed indicator $S(L)$ reflects the relative linguistic disadvantage of L-monolinguals in terms of socio-economic standing, public institutions, politics, media and community support. Througout the paper we simply write S for $S(L)$.

Summarizing, in Giles *et al.* (1977) the authors propose three categories of factors that influence language vitality: demographic factors, institutional support and status factors. The linguistic composition of the population X and the linguistic concentration measure C are the *demographic* factors considered in the present model. The status S and the education parameters s_{LR_1,LR_2} reflect important aspects of institutional support and status factors in the Giles *et al.* (1977) framework.

3.4.2 Effects of the Linguistic Environment on Transmission and Acquisition

Next, we propose simplifying assumptions on how the linguistic composition as well as the environment parameters affect the four quantities ψ_F, q_{LR} s_{LR_1,LR_2} and p_{LR_1,LR_2} driving the language dynamics.

Family formation: The expected number of families of a certain type F depends on the linguistic composition of the population X as well as on linguistic concentration C. Hence, $\psi(F) = \psi(F; C, X)$. The higher the linguistic concentration, the lower the expected number of mixed families.

Language transmission: Recall, $q_{LR}(F)$ denotes the fraction of families of type F bringing up children with language repertoire LR. Before stating hypotheses on how these quantities depend on our parameters describing the linguistic environment, we make some simplifying assumptions:

A0: The fraction $q_{LR}(F)$ depends on the status variable, linguistic concentration and the linguistic composition of the population, i.e. $q_{LR}(F) = q_{LR}(F; C, S, X)$.

A1: All children in one family have same language repertoire

A2: Both parents shall be able to communicate with their children, i.e. $q_L(HH) \equiv q_L(HB) \equiv 0$ and $q_H(LL) \equiv q_H(LB) \equiv 0$

A3: The more present a language l is in a certain family type, the more likely it is that children become monolinguals in l:

A3a: $q_H(HH; C, S, X) \geq q_H(HB; C, S, X) \geq q_H(BB; C, S, X)$

A3b: $q_L(LL; C, S, X) \geq q_L(LB; C, S, X) \geq q_L(BB; C, S, X)$

A4: The higher the number of l-monoglots, the higher the incentive to transmit l to ones children (Cf. Wickström (2005) for a justification):

A4a: If X_H increases, then q_H and q_B do not decrease and q_L does not increase

A4b: If X_L increases, then q_L and q_B do not decrease and q_H does not increase

A5: In an HH family children grow up as monolingual H speakers, i.e. $q_H(HH) \equiv 1$

Furthermore, we assume that the $q_{LR}(F; C, S, X)$ are continuous and almost everywhere differentiable functions with respect to X. Most of these assumption can already been found in Wickström (2005) and Templin *et al.* (2016). Let us proceed with the effect linguistic environment parameters have on the q-fractions.

The status parameters $S_{SE}(L)$ and $S_{OF}(L)$ reflect the relative disadvantage of L-monolinguals. The lower both parameters, the higher the disadvantage in socio-economic and institutional terms, and, in turn, the lower the utility from speaking the minority language. Consequently, the lower both parameters, the lower the incentive – from an instrumental point of view – to transmit L to the next generation. Furthermore, families with low SES have less resources they can use for successful transmission. A third effect of $S_{SE}(L)$ and $S_{OF}(L)$ on the transmission of L mentioned earlier has to do with the prestige of L. The higher the average socio-economic status and the more official domains the language can be used for, the higher might be it's symbolic status. Note that all three effects push in the same direction. Therefore, we assume that the higher any of the two status variables, the lower probability that HB or BB families only transmit H. Similar arguments can be put forward for LL and LB families. The lower $S_{SE}(L)$ and $S_{OF}(L)$, the higher the incentive to transmit or teach (e.g. in kindergarten or preschool) the majority language to children. Recall, $S = S(L)$ is a composed status indicator comprising the socio-economic as well as the institutional dimension. In view of the above considerations, we assume that

$$\frac{\partial q_H(HB; C, S, X)}{\partial S}, \frac{\partial q_H(BB; C, S, X)}{\partial S} \leq 0, \tag{2}$$

$$\frac{\partial q_L(LL; C, S, X)}{\partial S} \frac{\partial q_L(LB; C, S, X)}{\partial S} \geq 0, \tag{3}$$

Note, that due to assumptions A2 and A3a we have $q_B(HB) = 1 - q_H(HB)$, $q_B(LL) = 1 - q_L(LL)$ and $q_B(LB) = 1 - q_L(LB)$. Plugging this into (2) and (3) yields the respective statements for $q_B(HB)$, $q_B(LL)$ and $q_B(LB)$. Furthermore, $q_L(BB) + q_B(BB) = 1 - q_H(BB)$. Hence,

$$\frac{\partial q_B(BB; C, S, X)}{\partial S} + \frac{\partial q_L(BB; C, S, X)}{\partial S} \geq 0. \tag{4}$$

Linguistic concentration affects the exposure to both languages. Strong linguistic concentration keeps the expose to H low for L-speaking children and vice versa. Since bilinguals are considered rather as L's than as H's, we assume that given high concentration LB and BB families predominantly live in L dominated neighborhoods. HB families are assumed to be found in L-dominated as well as H-dominated neighborhoods. Therefore,

$$\frac{\partial q_L(LL;C,S,X)}{\partial C}, \frac{\partial q_L(LB;C,S,X)}{\partial C} \geq 0, \qquad \frac{\partial q_H(BB;C,S,X)}{\partial C} \leq 0. \tag{5}$$

Due to the last statement we also get $\partial q_B(BB;C,S,X)/\partial C + \partial q_L(BB;C,S,X)/\partial C \geq 0$.

Language education: We assume language learning at school to depend only on external policy factors. Hence, the s_{LR_1,LR_2} are conceptualized as constant model parameters. They do not depend on the distribution of speakers but reflect language education policies and the quality of language education.

We assume that neither H-monolingual nor bilingual children will unlearn H through formal education, i.e. $s_{H,L} = s_{B,L} = 0$. Hence, $s_{H,H} = 1 - s_{H,B}$ and $s_{B,B} = 1 - s_{B,H}$. Note, if $s_{H,B} > 0$, then some former H-monolingual children learn L at school. This is only possible if L is taught as a second language. If $s_{B,H} > 0$, then some children entering school as bilinguals leave the school system as H-monolinguals, which might be the case if a H-only language policy is applied. In principle we could include the option that L-monolingual children can get educated fully in L, which translates to $s_{L,L} > 0$. In practice, since H is a dominant majority language, all children learn H at some point in their educational career. Therefore, we set $s_{L,L} = 0$. Hence, $s_{L,B} = 1 - s_{L,H}$. Finally, it is reasonable to assume that bilingual children are more likely to leave school as H-monolinguals than children entering school as L-monolinguals, i.e. $s_{B,H} > s_{L,H}$.

Adult language learning: The acquisition of the local language H by adult newcomers is a question of exposure, efficiency, (economic) incentives and costs. The same approach can be applied to H-monoglots learning the minority language L. The quantities describing adult language learning in the model are $a_{L,B}$ (L-monoglots become bilingual) and $a_{H,B}$ (H-monoglots become bilingual). Since linguistic concentration C has a strong effect on exposure to the other language, we assume that both $a_{L,B}$ and $a_{H,B}$ decrease if C increases. The role of the status variable S is not as easy to pin down. On the one hand, if the economic and official statuses of the minority language group are low, then there are strong (economic) incentive for L-monoglots to learn H, since H promises much better economic opportunities. On the other hand, if the socio-economic status is low, then L-monoglots might have only few financial and educational resources, a potential hindrance for learning a new language (efficiency and costs). Appropriate programs encouraging and supporting the acquisition of the local language (e.g. offering free language classes) might be able to ease the socio-economic disadvantages. Later on, when specific functional expressions are introduced for $a_{L,B}$ and $a_{H,B}$, both effects are taken into account. The support of the local language acquisition is reflected in one parameter of the functional expressions.

4 General Model Formulation

In this section we state the formal language dynamics model. As before, X_H, X_L and X_B denote the fractions of H-monoglots, L-monoglots and bilinguals. Since the population size might not be constant we also work with absolute numbers. Let N_H be the number of H-monoglots, N_L be the number of L-monoglots and N_B be the number of bilinguals. The total population size is given by $N := N_H + N_L + N_B$. Note that $N_H = N \cdot X_H$, $N_L = N \cdot X_L$ and $N_B = N \cdot X_B$. We use both, relative numbers (X_H) and absolute numbers (N_H), since ψ and q only depend on relative numbers, while the overall dynamics are described in absolute terms. The language dynamics models describes in

mathematical terms how N_H, N_L and N_B change over time as a result of family formation, language transmission, language learning in formal education and adult language learning. Let $X := (X_H, X_L)$.

Family formation is the result of a random search and mating process affected by linguistic concentration C. As in Templin *et al.* (2016) we assume that both adults shall be able to communicate properly with one another. Hence, we neglect families of type HL. The family formation process yields the following distribution of family types ψ:

$$\psi(HH; C, X) = \big(C + (1-C)X_H\big)X_H + (1-C)X_H X_L \tag{6}$$

$$\psi(HB; C, X) = 2(1-C)X_H X_B \tag{7}$$

$$\psi(LL; C, X) = \left(1 + C\frac{X_H}{1-X_H}\right) X_L^2 + (1-C)X_H X_L \tag{8}$$

$$\psi(LB; C, X) = 2\left(1 + C\frac{X_H}{1-X_H}\right) X_L X_B \tag{9}$$

$$\psi(BB; C, X) = \left(1 + C\frac{X_H}{1-X_H}\right) X_B^2. \tag{10}$$

See the Appendix for a justification of the (6)-(10). Note, for $C = 0$, i.e. no linguistic concentration, the family type distribution is the same as given in Templin *et al.* (2016).

The process of language transmission from one generation to the next is determined by the distribution of family types and the fractions $q_{LR}(F)$. First of all, for all family types F these fractions have to add up to one. Given the distribution of language repertoires (X) in one generation and the relevant environment parameters (the status indicator $S(L)$ and the linguistic concentration C), the fraction of the next generation equipped with the language repertoire LR is given by

$$\sum_F q_{LR}(F; C, S, X) \cdot \psi(F; C, X). \tag{11}$$

In the above formula $q_{LR}(F; C, S, X) \cdot \psi(F; C, X)$ is the (relative) number of F-type families ($\psi(F)$) times the fraction of F-type families raising their children with repertoire LR ($q_{LR}(F)$). This is the summed over all family types F.

In the following let t be time measured in years. $N(t)$ is the overall population size at time t. $N_H(t)$, $N_L(t)$ and $N_B(t)$ denote the sizes of the language repertoire groups. By λ we denote the annual birth rate and by μ the annual death rate. Looking at empirical numbers, we observe that in several "developed countries" fertility rates that are not sufficiently high to preserve the population in the long-term. For example, in the EU the average number of children per women fluctuated around 1.5 in the last decades, while approximately 2.1 children are necessary to reproduce the population, see e.g. Eurostat (2014). Hence, we mostly have $\lambda < \mu$. It is assumed that birth and death rates are the same for all language repertoire groups. The model could easily be adjusted to cases of differing death and birth rates for the different language repertoire groups and family types. We define $X(t) = (X_H(t), X_L(t)) := (N_H(t), N_L(t))/N(t)$. The vector $X(t)$ describes the linguistic composition of the population at time t (relative instead of absolute numbers). Neglecting mobility, language education and adult language learning for the moment, the dynamics of the basic model can be described by the following three differential equations:

$$\dot{N}_{LR}(t) = -\mu N_{LR}(t) \; + \; \lambda N(t) \sum_F q_{LR}(F; C, S, X(t))\psi(F; C, X(t)), \tag{12}$$

$LR = H, L, B$. The first summand represents the number of people with language repertoire LR dying at time t. The second summand represents all the children raised with language repertoire LR at time

t. The overall population size changes according to $\dot{N}(t) = (\lambda - \mu)N(t)$. Note, to fully describe the dynamic system $\dot{N}(t)$, $\dot{N}_H(t)$ and $\dot{N}_L(t)$ are sufficient, since $N_C = N - N_H - N_L$. For better readability we introduce

$$f_{LR}(t) := \sum_F q_{LR}(F; C, S, X(t))\psi(F; C, X(t)), \qquad LR = H, L, B. \tag{13}$$

Next, we extend the basic model by stepwise including schooling, adult language learning and migration.

Formal education: Recall, s_{LR_1,LR_2} is the fraction of children entering school with repertoire LR_1 and leaving with LR_2. We assumed above that $s_{H,L}, s_{B,L}, s_{L,L} = 0$. For simplicity, we now also assume that there are no children entering school as L's and leaving as H's, i.e. $s_{L,H} = 0$. What is considered is the learning of an additional language ($L \to B$ and $H \to B$) as well as the loss of the minority language due to exclusive schooling in H ($B \to H$). The first extension of the basic model in (12) is

$$\dot{N}_H(t) = -\mu N_H(t) \; + \; \lambda N(t)\big((1 - s_{H,B})f_H(t) + s_{B,H}f_B(t)\big) \tag{14}$$

$$\dot{N}_L(t) = -\mu N_L(t) \; + \; \lambda N(t)(1 - s_{L,B})f_L(t). \tag{15}$$

To simplify notation even further we introduce

$$g_H(t) := (1 - s_{H,B})f_H(t) + s_{B,H}f_B(t), \tag{16}$$

$$g_L(t) := (1 - s_{L,B})f_L(t). \tag{17}$$

Adult language learning: We only consider acquisition of an additional language, i.e. the transition from monolingualism to bilingualsm ($a_{H,B}$ and $a_{L,B}$). Note, in contrast to the constants $c_{H,B}$ and $c_{L,B}$, the transition parameters $a_{H,B}$ and $a_{L,B}$ depend on C, $S(L)$ and X. Using the notation introduced in (16)-(17) we add one additional term in (14) and (15). Let $LR = H, L$. Writing $a_{LR,B}(t)$ for $a_{LR,B}(C, S, X(t))$, thee second extension is given by

$$\dot{N}_{LR}(t) = -\big[\mu + (1 - \mu)a_{LR,B}(t)\big]N_{LR}(t) \; + \; \lambda N(t)g_{LR}(t). \tag{18}$$

Mobility: In most of the language competition models reviewed above the population was considered to be constant in the sense that no new/external individuals enter the population (except the ones born within the population) and that individuals do not leave the territory. Here, we explicitly model an external inflow of new individuals. We focus on the migration of people with a heritage language that differs from the dominant or national language of the host country. In models with different growth rates for the different language groups, see below, external mobility can be incorporated into the growth parameters (Malthus rates), cf. Kandler *et al.* (2010). Here, external mobility is modeled more explicitly. The absolute number of people equipped with language repertoire LR migrating to the population at time t is denoted by $M_{LR}(t)$. The total number of migrants at time t is given by $M(t) := M_H(t) + M_L(t) + M_B(t)$. Note, in principle $M_{LR}(t)$ could be negative, which would indicate net emigration of LR's. Later we investigate two special cases. In the general case we get $\dot{N}(t) = (\lambda - \mu)N(t) + M(t)$ and hence, $LR = H, L$,

$$\dot{N}_{LR}(t) = -\big[\mu + (1 - \mu)a_{LR,B}(t)\big]N_{LR}(t) \; + \; \lambda N(t)g_{LR}(t) \; + \; M_{LR}(t). \tag{19}$$

Additionally, we define $m_{LR}(t) = M_{LR}(t)/M(t)$, the share of LR mobility. In the following paragrahps we consider two special cases for migration. In the first case, migration is assumed to be constant over time, i.e. every year the same absolute number of migrants enter the population (e.g. 100,000 migrants per year). In the second case, migration is constant relative to the population size. If e.g. the government

allows for an annual migration of 2% of the total population size, then migration is constant relative to the population size.

Special case 1 (Constant absolute migration flow): Let $M_{LR}(t) = M_{LR}$ be constant. Trivially, if $M = 0$ and $\lambda = \mu$ the total population size remains constant. Furthermore, if $M > 0$ and $\lambda < \mu$, then the population size $N(t)$ converges to the steady state $N(\infty) := M/(\mu - \lambda)$. Let $\lambda < \mu$ and $M > 0$. Applying the quotient rule to $X_{LR} = N_{LR}/N$ and setting $N = N(\infty)$ we obtain for the steady state population size

$$\dot{X}_{LR}(t) = -\big[\mu + (1-\mu)a_{LR,B}(t)\big]X_{LR}(t) \,+\, \lambda g_{LR}(t) \,+\, (\mu - \lambda)m_{LR}. \tag{20}$$

Special case 2 (Constant relative migration flow): Let $M(t)/N(t) =: \nu$ be constant as well as the fractions $M_{LR}(t)/N(t) =: \nu_{LR}$. Then, $\nu = \nu_H + \nu_L + \nu_B$, $\dot{N}(t) = (\lambda - \mu + \nu)N(t)$ and

$$\dot{N}_{LR}(t) = -\big[\mu + (1-\mu)a_{LR,B}(t)\big]N_{LR}(t) \,+\, \lambda N(t)g_{LR}(t) \,+\, \nu_{LR}N(t). \tag{21}$$

Applying the quotient rule to $X_{LR} = N_{LR}/N$ we obtain

$$\dot{X}_{LR}(t) = -(1-\mu)a_{LR,B}(t)X_{LR}(t) \,+\, \lambda g_{LR}(t) \,+\, \nu_{LR} - (\lambda + \nu)X_{LR}(t). \tag{22}$$

Note that $\nu_{LR} = M_{LR}/N = \nu M_{LR}/M = \nu m_{LR}$. If $\lambda < \mu$ and $(\mu - \lambda) = \nu$, i.e. if the total population size remains constant, then (20) and (22) yield the same formula.

5 Specific Functional Forms

In this section we specify the functions $q_{LR}(F; C, S; X)$ and $a_{LR,B}(C, S, X)$. For intergenerational language transmission we build on the functional expressions proposed in Templin *et al.* (2016). Here, these functional expressions are denoted by $\widetilde{q}_L(F)$. For non-negative parameters $0 < \beta < \delta$ and $\varepsilon + \gamma < \zeta < 1$ consider the functions

$$\begin{aligned}
\widetilde{q}_H(HB; S, X) &:= \max\{0, \zeta S(H) - \beta S(L)X_L\} \\
\widetilde{q}_H(BB; S, X) &:= \max\{0, \varepsilon S(H) + \gamma S(H)X_H - \delta S(L)X_L\}
\end{aligned}$$

To get the respective formulas for$\widetilde{q}_L(LB)$ and $\widetilde{q}_L(LB)$ we just exchange roles of H and L in the above formulae, cf. Templin *et al.* (2016). Note that $S(L) = 1 - S(H)$. These functions are constructed in such a way that if X_H is sufficiently high and $S(L)$ is sufficiently low, then bilingual (BB) and even mixed couples (LB) do not raise their children as L-monoglots. Earlier we made the assumption that $q_H(HH) = 1$. To preserve symmetry, we now also assume that $q_L(LL) = 1$, i.e. $\eta = 0$. Both assumptions imply that parents can only transmit languages they speak properly.

In Templin *et al.* (2016) the authors neglected linguistic concentration, which corresponds to $C = 0$. Therefore, we set $q_{LR}(F; C, S, X) = \widetilde{q}_{LR}(F; S, X)$ whenever $C = 0$. For $C = 1$, i.e. maximal linguistic concentration, we assume that LB and BB families predominantly live in L-dominated areas. Therefore, given $C = 1$, children in BB families do not grow up as H-monoglots, and children in LB families become L-monoglots. Furthermore, we assume that half of the HB families live in H-dominated areas, while the other half resides in L-dominated areas. Given maximal concentration, we thus assume that one half of all HB children grow up as H-monoglots, while the other half becomes

bilingual. This yields $q_H(HH) = q_L(LL) \equiv 1$ and

$$\begin{aligned} q_H(HB; C, S, X) &= (1-C) \cdot \widetilde{q}_H(HB; S, X) + C/2 \\ q_H(BB; C, S, X) &= (1-C) \cdot \widetilde{q}_H(BB; S, X) \\ q_L(LB; C, S, X) &= (1-C) \cdot \widetilde{q}_L(LB; S, X) + C \\ q_L(BB; C, S, X) &= (1-C) \cdot \widetilde{q}_L(BB; S, X) + C. \end{aligned}$$

Next, we specify the functions $a_{l,B}(S(L), C; X)$, $l = H, L$. Let θ and ϕ be a parameters between 0 and 1. Support for the acquisition of H and L for monolingual speakers of the other language is modeled by the parameters u_H and u_L. We set

$$\begin{aligned} \tilde{a}_{L,B} &:= \max\left\{0, \theta(1-S)X_H - \phi(1-u_H)\right\}, \\ \tilde{a}_{H,B} &:= \max\left\{0, \theta S X_L - \phi(1-u_L)\right\}. \end{aligned}$$

For maximal support for the acquisition of the local language H for L-monoglots, i.e. $u_H = 1$, the function $\tilde{a}_{L,B}$ is strictly increasing in $S(H)$ and X_H. This reflects that the higher the socio-economic and communicational incentives associated with H as well as the exposure to H, the more L-monolinguals learn H. If no support is made available, then $S(H)$ and X_H have to be high enough such that L-monoglots acquire H. Since linguistic concentration hinders learning of the other language, we set $a_{l,B} = (1 - C)\, \tilde{a}_{l,B}$, $l = H, L$. Hence, given maximal concentration/segregation, no monolingual individuals learn the other language.

6 Steady States

In this section we want to investigate possible steady states of the systems described in the previous two sections. First, we will look at monolingual steady states, that is situations without any bilingual speakers. Thereafter we are concerned with the more interesting case of steady states containing a bilingual subpopulation. Steady states are denoted by $\hat{X}_H, \hat{X}_L$ and $\hat{X}_B$. We only consider the special cases with constant absolute and constant relative migration flows and assume that $\lambda \leq \mu$.

6.1 Monolingual steady states

In a monolingual steady state we have $\hat{X}_B = 0$ and hence $\hat{X}_H + \hat{X}_L = 1$. Clearly, this can only yield a steady state if no bilingual speakers enter the society any more, i.e. $M_B \equiv 0$. Moreover, neither children nor adults of either language group can learn the other language, otherwise some would become bilingual. Since HL couples have probability zero, all families are of types HH and LL. This implies that a monolingual steady state is only possible if either there is only one language group left ($\hat{X}_H = 1$ or $\hat{X}_L = 1$,) or the population is entirely segregated, which would include L-only eduction for L-pupils and H-only education for H-pupils. For $\hat{X}_H = 1$ to be stable there can not be any L-migration and for $\hat{X}_L = 1$ to be stable (economic) incentives to learn H have to be sufficiently low. We are predominantly interested in cases with a positive inflow of L-speaking newcomers and where the status of the locally dominant language H is higher than the one of L, i.e. $S(L) < 1/2$. Therefore, we neglect the cases for which monolingual equilibria exist.

6.2 Bilingual steady states

Steady states without any monolingual speakers of the minority language, i.e. $\hat{X}_L = 0$, are only possible, if all migrants already enter the society as H-monoglots or bilinguals ($M_L \equiv 0$). Given a highly restrictive migration regime, this could be an imaginable scenario. In reality, though, most states face an influx of at least some people with no or only little knowledge of the locally dominant language. For the numerical examples in the subsequent section we therefore assume $M_L > 0$.

In principal, steady states without a monolingual H-subpopulation ($\hat{X}_H = 0$) are possible as well. Vast migration of L-monoglots combined with a low fertility rate (small λ) and strong linguistic concentration can lead to a situation in which low rates of H-acquisition by L's can not compensate for the shrinking monolingual H population. Whether or not then part of the population remains bilingual, i.e. whether this yields a steady state with $\hat{X}_B > 0$, depends on the status of H. If the status is relatively low, it is theoretically possible that H is not learned or transmitted anymore, such that $\hat{X}_L = 1$, but this is a very unlikely scenario.

In a third possible type of steady states there are monolingual speakers of both languages as well as bilinguals, i.e. $\hat{X}_H, \hat{X}_L, \hat{X}_B > 0$. Given a steady influx of L-monolinguals combined with low linguistic concentration and low status of L, a sufficient number of HB and BB couples raise their children only in language H, such that a certain H-monolingual subpopulation remains stable. In the following section we present numerical examples with and without such an H-monolingual subpopulation.

7 Numerical Examples

The aim of this section is to provide a few numerical examples for the model described above and to illustrate the language competition graphically. We restrict ourselves to the special case of constant absolute migration flow and assume that all newcomers enter the population as L-monoglots ($m_L = 1$).

For intergenerational language transmission, i.e. for the functions $q_{LR}(F)$, we use the same parameter constellation as in the numerical examples of Templin *et al.* (2016): $\zeta = 0.8$, $\varepsilon = 0.4$, $\beta = 0.4$, $\delta = 0.7$, $\gamma = 0.1$. The yearly birth rate is assumed to be at 1.5% ($\lambda = 0.015$), while the yearly death rate is at 2% ($\mu = 0.02$). For adult language learning we set $\theta = 0.3$ and $\phi = 0.05$. Furthermore, we assume that there is no support for the acquisition of the newcomer language L by monolingual speakers of the dominant language, i.e. $u_L = 0$. Similarly, for the few numerical examples investigated below it is assumed that H-monolingual pupils do not learn L at school, i.e. $s_{H,B} = 0$. Setting $u_L = 0$ or $s_{H,B} = 0$ would have a negative effect on the relative number of H-monoglots.

Hence, for the numerical examples only four parameters are varying: the status variable $S(L)$, the linguistic concentration measure C, the fraction of pupils entering the school as L-monoglots and additionally learn H during formal education and the support for the acquisition of the locally dominant language H by newcomers u_H.

In Table 1 parameter values for six numerical examples are specified. Additionally, the corresponding steady states are provided. Table 2 displays the distribution of speakers after 50 and 100 years, starting at $X_H(0) = 0.85$ and $X_L(0) = 0.15$ and at the steady state population size. We choose the status of the minority language $S(L)$ in the first four examples to be 0.2, while for the last two examples the status is 0.4 and hence considerably higher. In most examples the linguistic concentration measure C is at 1/2, 80% of all L-monolingual children learn the dominant language H at school and maximal support for the acquisition of the dominant language by adults is provided ($u_H = 1$). For Example 1, which shall function as a reference case, the steady states consists of 25% H-monoglots, 12% L-Monoglots

	$S(L)$	C	$s_{L,B}$	u_H	$\hat{X}_H$	$\hat{X}_L$	$\hat{X}_B$
Example 1	0.2	0.5	0.8	1	0.252	0.122	0.626
Example 2	0.2	0.5	0.8	0	0.126	0.327	0.547
Example 3	0.2	0	0.8	1	0.456	0.040	0.504
Example 4	0.2	1	0.8	1	0	0.4	0.6
Example 5	0.4	0.5	0.8	1	0.128	0.210	0.662
Example 6	0.4	0.5	0.5	1	0.060	0.385	0.554

Table 1: Parameter values for six numerical examples and corresponding steady states.

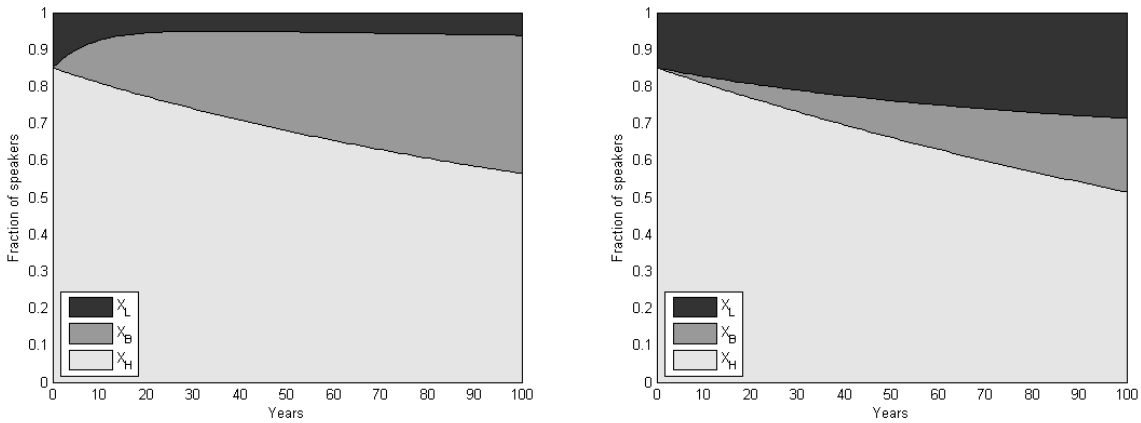
and 63% bilinguals. Comparing the steady states with the first line of Table 2 it becomes apparent that even after 100 years the linguistic composition is not close to its steady state yet. In Example 2 no support is provided for the acquisition of H by L-monolingual adults. As expected this increases the number of L-monolgots in the steady state by 20%, while after 100 years we only observe an increase of 2.5%. Example 3 shows results for zero linguistic concentration. Due to high rates of exposure to H and extensive contact with H-monolgots, sufficiently many HB families emerge and a relatively large number of children in those and BB families are raised as H-monolgots. This yields a steady state in which almost half of the population is monolingual in H. In contrast, Example 4 illustrates the case of full linguistic concentration. For $C = 1$ no HB couples form and none of the BB families raise their children only in language H. As a result, since $\lambda < \mu$, the H-monolingual population dies out in the long run. The higher status of language L in Example 5 yields, compared with Example 1, a decrease in the number of H-monoglots and an increase in the number of L-monoglots. This can be explained by the higher attractiveness of language L and, respectively, by the lower attractiveness of H. Fewer L-monoglots become bilingual and, at the same time, fewer bilingual parents only transmit H to their children. In the last example only half of all L-monolingual pupils learn H at school. This results in a even lower number of H-monolinguals and an increased number of L-monolingual adults.

Looking at the numbers in Table 2 and comparing them with the steady states in Table 1 two important properties of the model can be observed. First, one can see that it takes many years until the linguistic composition reaches its steady state. Hence, for the analysis of short- and medium-term developments – for which the parameters can be assumed to be constant – it is not sufficient just to consider steady states. Second, one can see that in the short- and medium-term a change in one or more parameters mainly affects the fraction of L-monolingual and bilingual speakers. This can be explained by the fact that few H-monolinguals learn L despite of its status. Furthermore, $s_{L,B}$ and u_H only affect L's. On top of that it takes some generations for intergeneration language transmission and for low fertility to change the linguistic composition noticeably.

Last, we provide a few trajectories that shall graphically illustrate the results depicted in Tables 1 and 2. In Figure 1 the Examples 1 and 4 are displayed for initial composition $X_H(0) = 0.85$ and $X_L(0) = 0.15$ and for steady state population size. Figure 2 shows trajectories for the same examples with the same initial composition but with changing population size. Here, an initial population of 100 million people and a constant yearly migration inflow of 200,000 L-monolinguals is considered. Compared with the situation in which the total population size is at its steady state (40 million), here the fractions of L-monolgots and bilinguals are increasing much slower. To get a better understanding of how the different language groups evolve in the medium-term, Figure 3 shows the absolute number of speakers for Examples 1 and 4. It can be seen that although the fraction of H-monoglots decreases only slightly (see Figure 2), the absolute number of H-monoglots and with them the overall population size shrinks dramatically over the first 100 years due to the low fertility rate.

	$X_H(50)$	$X_L(50)$	$X_B(50)$	$X_H(100)$	$X_L(100)$	$X_B(100)$
Example 1	0.681	0.052	0.267	0.562	0.062	0.376
Example 2	0.679	0.069	0.252	0.557	0.085	0.357
Example 3	0.702	0.027	0.271	0.610	0.030	0.360
Example 4	0.664	0.237	0.099	0.517	0.286	0.197
Example 5	0.670	0.066	0.264.	0.536	0.079	0.385
Example 6	0.670	0.074	0.256	0.534	0.092	0.374

Table 2: Distribution of speakers after 50 and 100 years for the numerical examples defined in Table 1. The initial linguistic composition is $X_H(0) = 0.85$, $X_L(0) = 0.15$ and $X_B(0) = 0$ and the population size is assumed to be in the steady state.



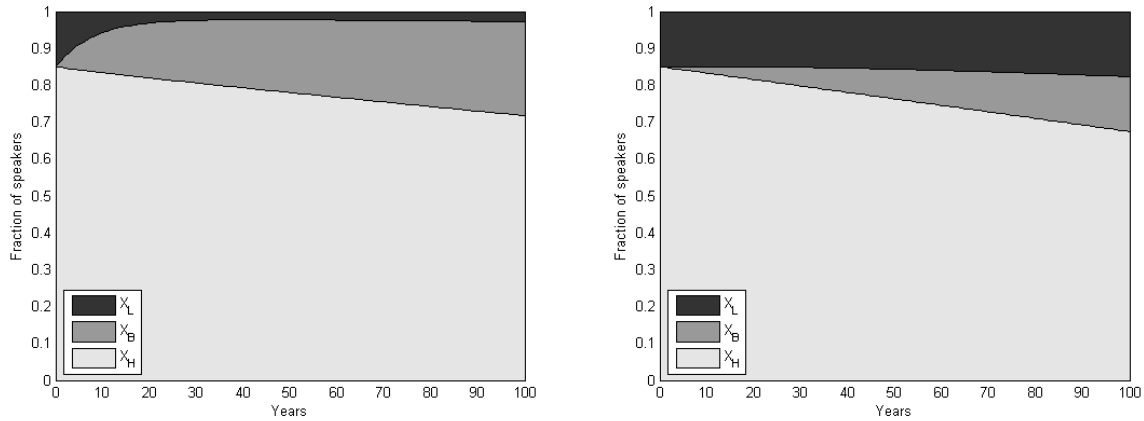
(a) Example 1

(b) Example 4

Figure 1: Trajectories for fractions of speakers for Examples 1 and 4 for initial composition $X_H(0) = 0.85$ and $X_L(0) = 0.15$ and for steady state population size.

8 Conclusion and Outlook

In this paper we develop and analyze a language dynamics model for "new minorities" that includes intergenerational language transmission, formal language education and adult language learning. Speakers of the newcomer language L enter a society with one (main) official language H, which is spoken by most society members. To improve their socio-economic position within the host country, some of the newcomers learn H. The model includes a parameter measuring linguistic concentration. The more segregated both language groups are, the lower is the exposure to the other language and thus fewer people become bilingual. More important for longer-term language dynamics than adult language learning is intergenerational language transmission and language education for the descendants of newcomers. Individuals form families and transmit one or all of their languages to their children. The decisions on which languages to transmit are driven by instrumental and emotional motives, as well as costs. While the locally dominant language usually promises better socio-economic opportunities for their children, many newcomers want to transmit their heritage language to the next generation. Especially if children live in a social environment dominated by H and if they are only educated in H, then transmitting the minority language might not be successful and after two or three generations the language is lost – a well known empirical observation in socio-linguistic studies. Therefore, linguistic concentration supports the



(a) Example 1

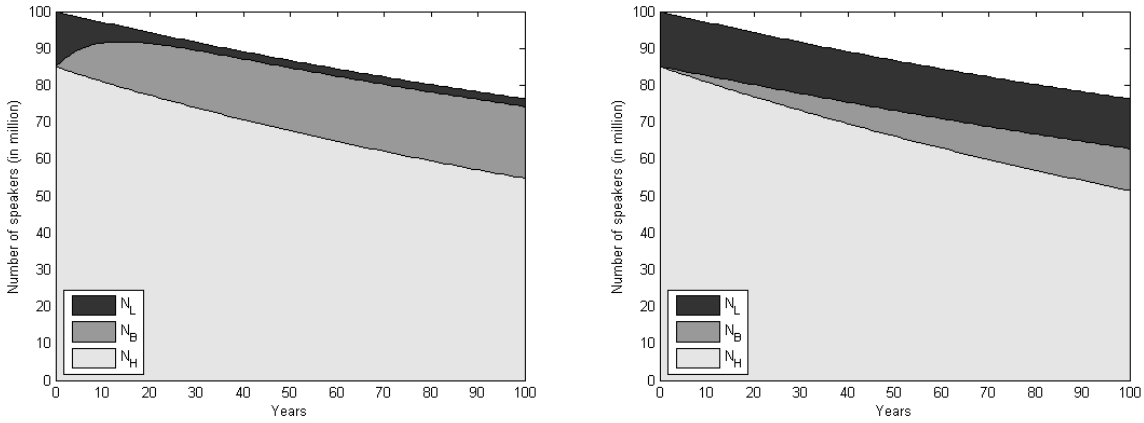
(b) Example 4

Figure 2: Trajectories for fractions of speakers for Examples 1 and 4. The initial population consists of 100 million people (85 million are H-monoglots and 15 million are L-monolgots) and every year 200,000 new L-monolinguals enter the population.

transmission of L, but might hinder a proper acquisition of H.

Regarding the abstract status parameter used in a number of language competition models, we share the criticism expressed by Fernando *et al.* (2010) and others. At the same time, we emphasize the role of factors related to the socio-economic and the institutional status for individual language related decisions. So instead of omitting the status from the model, we suggest a measurable status parameter composed of the socio-economic status of the language repertoire groups and the institutional status, which is operationalized as the number of official domains the languages can be used for. This yields an indicator of (1) the socio-economic opportunities related to a language, (2) the usefulness of a language in communication with state authorities and (3) the socio-economic resources available to members of a language repertoire group. Furthermore, the model accounts for population dynamics. Using birth, death and migration rates obtained from empirical surveys allows for the model to be closer to real developments.

The analysis of long-term dynamics shows that monolingual steady states can only occur for fully segregated populations. In such a scenario the majority of newcomers does not acquire the local language H, while the H-monolingual population constantly shrinks because of the low fertility rate until only speakers of L are left. Due to the relatively high status of H, such a monolingual steady state with only L's is rather unlikely. Much more likely are, however, steady states with monolinguals of both languages as well as bilinguals. This can be seen in Table 1, where steady states are calculated for different parameter constellations characterizing the linguistic environment. In all cases, except for Example 4, for which the H-monolingual population dies out in the long-run as a result of linguistic concentration and low fertility rates, the steady state consists of monolinguals of both languages as well as bilinguals. Table 2, in which the linguistic composition after 50 and 100 years is depicted for all the numerical examples, illustrates the insufficiency of an analysis concentrating only on steady states. Although steady states differ notably for the six examples, the difference after 50 and even 100 years is not as dramatic. It can be seen that during the first 50 years the H-monolingual population is barely affected by language policy measures targeting speakers of L. Since the "original" H-monolingual population dies out (low fertility) and "new" H-speakers have L-descendants, effects of such policies on X_H can only be observed in the long-run. For the fractions of L-monolinguals and bilinguals the short- and medium-term effects of



(a) Example 1

(b) Example 4

Figure 3: Trajectories for absolute number of speakers for Examples 1 and 4. The initial population consists of 100 million people (85 million are H-monoglots and 15 million are L-monolgots) and every year 200,000 new L-monolinguals enter the population.

changes in relevant model parameters are more significant. Here, in particular, the role of linguistic concentration must be emphasized. For maximal linguistic concentration ($C = 1$) after 50 years the fraction of L-monoglots is almost five times higher compared with the case of moderate linguistic concentration ($C = 0.5$). The effects of changes in model parameters related to language education and support for adult language learning are not as strong but can be observed as well. The same holds true for changes in the status parameter. In Section 7 we furthermore illustrate that neglecting population dynamics and assuming a steady state population size might have a strong impact on the analysis of short- and medium-term developments. If the overall population size is shrinking towards its steady state, then the influx of L's has a weaker effect on the linguistic composition of the population compared to the steady state case. At the same time, the absolute number of H-monolgots shrinks rapidly, while their relative number shrinks much slower. This shows that relative as well as absolute numbers should be considered in order to describe fully the population's linguistic development.

We see this paper as a point of departure for future research on language dynamics in societies with new language minorities. To get closer to the complexity of the processes at hand, the model can be extended and refined in several ways. First, in the current version of the model we only consider one language minority, while normally one can observe a variety of minority language groups. In the case of multiple linguistic minorities the locally dominant language H is not only a mean of upward social mobility, but can also function as a vehicular language used in communication between speakers of different minority languages. Second, different levels of linguistic competences or actual language use (instead of proficiency) could be modeled. Third, the current model concentrates on permanent migration and mostly neglects mobile individuals which stay only temporarily and leave the country after a couple of years. Particularly in view of highly skilled workers or international students, who frequently belong to this category, a model taking into account international vehicular languages that are (partially) spoken by locals as well as newcomers – as in the case of English – would be of interest.

Notes

[1]A set of differential equations that are used in evolutionary biology to describe the dynamics of biological systems in which two species compete with each other, normally one as a predator and one as a prey.

[2]Here, we use the term 'linguistic repertoire' in the very narrow sense as the set of languages a person commands, while only the two languages H and L are taken into account.

[3] Linguistic environment is a "[t]heoretical construct used for analytical purposes. It subsumes in an extensive (but obviously not exhaustive) fashion all the relevant information about the status, in the broadest sense of the word, of the various languages present in a given polity at a certain time. This includes the number of speakers, individual proficiency levels in the various languages, the domains of use of each language by different types of actors (individuals, corporations, the state, civil society organizations), and their attitudes towards the languages considered" (Grin & Vaillancourt, 1997, p. 49).

[4]For a more detailed typology (10 types) see Baker (2011, Chapter 10).

[5]"For all practical purposes, though, it seems realistic to expect that most parents will enroll their children in the local monolingual majority language school" (Caldas, 2012, p. 359).

[6]We can not go into great detail here. For extensive overviews on the connections between language and economics see, for example, Grin (2003) and Zhang & Grenier (2013).

[7]Note, for minority languages learned by a large number of H-monolinguals, such as English, the model has to be adjusted.

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A Appendix

Consider a population of size N, where N is large. We assume that half of the population is female and that the distribution of language repertoires is identical forboth sexes. Then, $N_{LR}/2$ is the number of female individuals with language repertoire LR. The distribution of repertoires is given by $X_H = N_H/N$, $X_L = N_L/N$ and $X_B = N_B/N$.

Family formation is conceptualized as a repeated random procedure of choosing pairs. Let us start with one such pair consisting of a female Y and a male Z. The probabilities that a certain pair is chosen depend on the numbers of speakers as well as on linguistic concentration. For maximal concentration, i.e. $C = 1$, H monolinguals only meet other H monolinguals and speakers of L only meet other speakers of L. For $C = 0$ meeting probabilities only depend on the sizes of the language groups. In between these two boundary cases we simply use linear interpolation. First, we derive the relevant conditional probabilities. In terms of conditional probability $C = 0$ translates to stochastic independence of Y and Z. Let $LR, LR' \in \{L, B\}$. We obtain

$$\mathbb{P}[Z = H|Y = H] = C + (1 - C)\mathbb{P}[Z = H] = C + (1 - C)X_H \tag{23}$$

$$\mathbb{P}[Z = LR|Y = H] = (1 - C)\mathbb{P}[Z = LR] \qquad = (1 - C)X_{LR} \tag{24}$$

If both speak the minority language, things are slightly more complicated. For $C = 0$ we have $\mathbb{P}[Z = LR|Y = LR'] = \mathbb{P}[Z = LR] = X_{LR}$. For $C = 1$ we get $\mathbb{P}[Z = LR|Y = LR'] = \mathbb{P}[Z = LR]/\mathbb{P}[Z = L, B] = X_{LR}/(1 - X_H)$. Linear interpolation yields

$$\mathbb{P}[Z = LR|Y = LR'] = X_{LR}\left(1 + C\frac{X_H}{1 - X_H}\right) \tag{25}$$

Next, we use that $\mathbb{P}[Y = LR_1, Z = LR_2] = \mathbb{P}[Y = LR_1]\mathbb{P}[Z = LR_2|Y = LR_1]$ and that $\mathbb{P}[Y = LR_1] = X_{LR_1}$, LR_1 and LR_2 being any of the three language repertoires H, L or B. Note that a family type LR_1LR_2, $LR_1 \neq LR_2$, is obtained either by $Y = LR_1$, $Z = LR_2$ or by $Y = LR_2$; $Z = LR_1$ (for the family type we do not take the gender of the parents into account). Hence, the probability to obtain a pair of type LR_1LR_2, denoted by $\mathbb{P}[LR_1LR_2]$ is given by $\mathbb{P}[Y = LR_1, Z = LR_2]$, if $LR_1 = L_2'$, and by $2\mathbb{P}[Y = LR_1, Z = LR_2]$, if $LR_1 \neq LR_2$.

All $N/2$ pairs are chosen randomly after one another. The total expected number of $LRLR'$-type pairs equals $N/2 \cdot \mathbb{P}[LR_1LR_2]$ and hence the expected fraction of LR_1LR_2-type pairs is $\mathbb{P}[LR_1LR_2]$. We assume that all couples of types other than HL form families. Recall, we assume that parents shall be able to properly communicate with each other, and therefore we exclude HL families. Hence, after this first step we have $N/2$ pairs with $N(1 - C)X_HX_L$ of them being of type HL. Splitting these HL pairs again and repeating the random selection we obtain new pairs of types HH, LL and HL. This procedure is repeated until only HH and LL pairs remain. This way, half of the $N(1 - C)X_HX_L$ pairs of type HL will be transformed into HH pairs, while the other half will form LL pairs. As a result, we obtain the numbers presented in equations (6)-(10). Note, due to the law of large numbers (N is assumed to be large), the realized number of LR_1LR_2-type of pairs can be approximated by the expected number.