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Interethnic relations, informal trading networks, and social integration: Imitation, habits, and social evolution*

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Abstract

The ethnically homogeneous middleman groups (EHMGs), which are informal trading net-works, are ubiquitous in less-developed economies where the legal infrastructure for contract enforcement is not well developed. This paper develops a formal model of social interaction among members of the EHMG as well as of social interaction in more general situations in a society consisting of identifiable ethnic or linguistic groups

Behavioral patterns are transmitted between generations and altered via imitation in social contacts. The model demonstrates how different discriminatory behavioral patterns can evolve and persist over time.

One result is that the trust between such groups can increase due to a higher frequency of inter-group contacts.

Keywords: cultural transmission units, ethnic networks, inter-ethnic interaction, imitation, identity, trust

JEL classification: E71, B52, J15, Z13

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1 INTRODUCTION

The ethnically homogeneous middleman groups (EHMGs), which are informal trading networks, are ubiquitous in less developed economies where the legal infrastructure for contract enforcement is not well developed. Classic examples of these informal trading networks are the ethnic or religions networks established by Chinese traders in Southeast Asia, the East Indian traders in East Africa, the Lebanese traders in North Africa, the Syrians in West Africa, the Jews in medieval Europe, etc. Landa (1981), Carr and Landa (1983) (reprinted in Landa, 1994), and Landa (2016) have developed a theory of the EHMG as a low-cost club-like institutional arrangement, alternative to contract law and vertically integrated firms, for the enforcement of contracts in environments characterized by contract uncertainty. Crucial to the theory is the idea that under conditions of contract uncertainty, a trader will choose to trade with partners with whom he/she has a particular tie, such as kinship or ethnicity. This is because codes of ethics or social norms are embedded in kinship or ethnic ties which promote mutual trust, and hence facilitate the enforcement of contracts. Furthermore, social sanctions exist to punish those who violate the norms of behavior governing kinship/ethnic relations. The aggregate effects of individual traders' discriminatory choice of trading partners along kinship or ethnic lines, is the emergence of the macro phenomenon of the EHMG, or ethnic trading network.

In this paper, we develop a model of social interaction between identifiable ethnic or religious groups. The point of departure is the theory of the EHMG, but we extend and generalize it to a wider scope of social situations, such as the treatment of ethnic or linguistic minorities or immigrant groups. One key feature is that group membership is observable, but not individual types. This opens up for different behavior towards individuals of different groups.[1] The behavior towards other people is modeled to be determined by upbringing, experience, and social norms.

This is done within a framework of imitation and limited rationality. Specifically, we assume that basic behavior is determined by a combination of upbringing and intentional choice influenced by experience (imitation) and social norms (behavioral dissonance).[2] That is, preferences formed by one generation of individuals (for example, merchants choosing kinsmen or fellow-ethnics as trading partners) are culturally transmitted through upbringing to the individuals of the following generation (sons of merchants), who then through their interaction with other individuals see the "inherited" habits reinforced or questioned. This experience in connection with the prevailing social norms as well as immediate individual benefits influences their possible decision to adopt a new behavior pattern. This pattern is then transmitted to the subsequent generations as the habits of the family, etc.

In contrast to many (economic) theories of evolution, the fitness of various types after social interactions is hence not an exogenous collection of parameters multiplying the frequency of each encounter, but is partially endogenously determined through conscious decisions of limited rationality. This approach is similar to that of Wickström (2005).[3]

The model of individual choice rests on the implicit assumption that behavioral patterns

[1] This aspect of our model is similar to models of statistical discrimination; see, for instance, Aigner and Cain (1977).

[2] In the language of evolutionary theory, we could say that behavior is determined by inheritance (of phenotype), environmental influence, and intentional choices.

[3] Of course, there exists a large literature on imitation theory; see, for instance, Accinelli, Brida, and Sánchez Carrera (2011) or – also for experimental evidence – Apesteguia, Huck, and Oechsler (2007).

formed at early stages of life are not altered at later ones independently of incentives facing the individual. That is, the first experiences in life and the then prevailing norms form the individual for the rest of its life, and this behavioral pattern is transferred as the *a priori* behavior to the individuals of next cohort. This hypothesis of *behavioral sclerosis* has some intuitive appeal and is the implicit assumption behind most of the discussion of the formation of social norms and evolutionary theory of games. Also the literature on investment in human capital, see for instance STIGLER and BECKER (1977), or the learning-by-doing literature can be viewed as supporting such a hypothesis.

2 THE EHMG AS A CULTURAL TRANSMISSION UNIT AND ITS EXTENSION TO SOCIAL INTEGRATION IN MODERN MULTI-ETHNIC SOCIETIES

LANDA (1991) developed a theory of the EHMG as a transmission unit. The relative success in cultural transmission – defined as the inheritance of determinants of behavior – depends on parents teaching their children and relatives, and individuals learning from or imitating the behavior of others, such as teachers, priests, or "big men". It is social learning that causes the communication of cultural traits directly from individual to individual, so that culture is a "population-level phenomenon". Cultural inheritance is important in the evolution of human behavior because it economizes on the costs of learning. Individuals, embedded in a cultural population, can learn to adapt to local condition by copying or imitating the everyday behavior of others in that cultural group. Copying or imitating soon becomes habitual and so the habit of behaving in a particular way will be reflected in the behavior of future generations of middleman-entrepreneurs. Thus, ethnocentrism of members of the EHMG is reinforced; those who are already in the middleman group will continue to prefer to choose trading partners from the younger generation belonging to the same kinship/ethnic group, thus excluding members of other ethnic groups. The cultural or ethnic boundaries of middleman trading networks thus persist over time.

This framework can, of course, be extended to cover other social phenomena than only ethnic trading patterns. With our model we try to embed the EHMG phenomena in a wider structure of interactions between identifiable social groups. Thereby we can throw some light on the persistence of many social structures and the slow or non-existent integration and inclusion of individuals from different background in the greater society. This, of course, can lead to discrimination and a lack of cohesion in society.

Our analysis can also illuminate the background of part of the resistance to globalizing tendencies that we can observe in today's world. This manifests itself for example as providing the ground for populist xenophobic movements in many European countries. Partially this seems to be a consequence of the free movement of labor as a core part of European integration and – probably even more relevant – a result of the refugee crisis. The societies become more and more heterogeneous. At the same time, there is very little social contact between individuals across groups. This seems to lead to situations where members of different groups distrust members of other groups, and we end up with a segmentation of society. The equilibria of our model can help explaining such phenomena.

3 A FORMAL MODEL OF CULTURAL TRANSMISSION

A basic assumptions underlying this work are that individual behavioral patterns are formed through imitation in the youth of an individual and thereafter remain largely unchanged. We operationalize this to mean that the behavioral characteristics of the parent is taken over by the child and retained unless the child at the beginning of life encounters a different behavioral pattern outside of the home. This interaction might cause the individual to change to a different behavior pattern.

3.1 BASIC STRUCTURE

Specifically, we limit ourselves to two distinct ethnic groups of people (referred to by the letters p and π in describing their behavioral characteristics; in general, we will adopt the convention of using Latin letters referring to one group and Greek letters referring to the other one) with whom each individual has to interact: the own group (Latin) and the others (Greek). Most of the analysis will be made from the point of view of the first group. The expressions pertaining to the other group can all be had by substituting Greek letters for the Latin ones and *vice versa*. Three possible behavioral patterns, types, are assumed:

type 1: trust all

type 2: trust only group members

type 3: trust nobody

By an encounter of two individuals the following payoffs are possible:

- (u_{tt}, u_{tt}) if both individuals trust one another,
- (u_{cc}, u_{cc}) if both distrust and try to cheat one another, and
- (u_{tc}, u_{ct}) if the first individual is trusting and the second one cheats the first one.

We assume the interaction to be of the prisoner's dilemma type:[4]

ASSUMPTION 3.1 $u_{ct} > u_{tt} > u_{cc} > u_{tc}$

The information structure of the model is given by the following assumption:

ASSUMPTION 3.2 The type of another person is not observable before an interaction, but its ethnicity is.

[4] This seems to us to be the natural assumption given the asymmetry of the situation. Note that the interactions are not in the form of a traditional coordination game. A simple coordination game (kissing a person on the right or left cheek when meeting) of course is symmetric and would have $u_{lr} = u_{rl}$ (in both cases a kiss on the mouth would result, which might be against accepted conventions: $u_{ll} = u_{rr} > u_{lr} = u_{rl}$). This does not capture the asymmetry of the situation. If I am gullible and cheated, I am bound to lose more than if I am prepared to be cheated. Similarly, if I cheat a trusting person, I should gain more than if I trust my opposite.

Letting p_i and π_i denote the fractions of the individuals of type i in the respective groups, denoting by r the probability that an individual from the first (L-)group encounters an individual from the same group (and by ρ the probability that a G−individual encounters a G−individual), and assuming that the behavioral patterns are distributed homogeneously in the two populations, we can write the expected utility of the encounters for an individual of each type:

$$\begin{aligned} EU_1 &= u_{tt}\left[r\left(p_1+p_2\right)+(1-r)\,\pi_1\right]+u_{tc}\left[rp_3+(1-r)\left(\pi_2+\pi_3\right)\right] \\ EU_2 &= u_{tt}r\left(p_1+p_2\right)+u_{ct}\,(1-r)\,\pi_1+u_{tc}rp_3+u_{cc}\,(1-r)\left(\pi_2+\pi_3\right) \\ EU_3 &= u_{ct}\left[r\left(p_1+p_2\right)+(1-r)\,\pi_1\right]+u_{cc}\left[rp_3+(1-r)\left(\pi_2+\pi_3\right)\right] \end{aligned} \quad (3.1)$$

It is readily seen that $EU_3 > EU_2 \geq EU_1$ for $r > 0$:

REMARK 3.1 The unique Nash equilibrium in dominant strategies in a one-shot game is to trust nobody, that is, $p_3 = \pi_3 = 1$ with an expected payoff for everyone equal to $EU_3^* = u_{cc}$.

This equilibrium is Pareto dominated by, for instance, a situation with $p_1 = \pi_1 = 1$, which for everyone has an expected payoff of $EU_1^* = u_{tt}$.

3.2 EVOLUTIONARY DYNAMICS AND BEHAVIORAL SCLEROSIS

The equilibrium in dominant strategies might not be relevant, though, if individual behavior is determined once and for all for each individual at the beginning of life. As noted above, we assume that the individual *a priori* in the first interaction with an individual outside the immediate family imitates the type of the parent (its "phenotype"), and then possibly adjusts its behavioral pattern due to the influences of the surroundings. This adjustment depends on the first such outside interaction and the prevailing social norm (*i.e.*, the environment), and the opportunity costs of having a certain behavioral pattern (which captures the intentional aspect of the behavior). For an individual from the L-group, the probability of a certain encounter depends on all the frequencies p_i's and π_i's, as well as r, the social norms within the ethnic group are given by the distribution of the p_i's, and the opportunity costs by the u_{ij}'s.

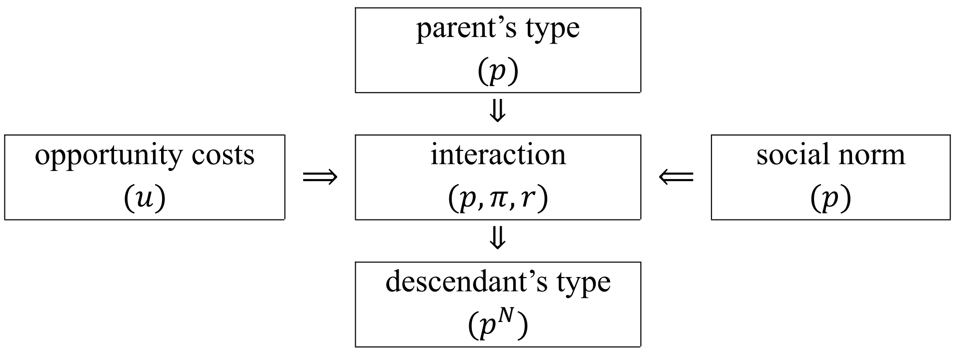

FIGURE 3.1 The dynamic structure of model

The basic structure of our model is schematically given in figure 3.1. We describe it with a set of functions f giving the transition probabilities between the individual types. For $i \neq k$, each $f^i_{kL_j}$ denotes the probability that the young L-individual of type k transforms (mutates) into an individual of type i after encountering an L-individual of type j, and similarly, $f^i_{kG_j}$ is the probability that a descendant of an L-individual of type k becomes an individual of type i after encountering a G-individual of type j. For $i = k$, we then find the probability that the individual retains its *a priori* type: $f^i_{iL_j}$ and $f^i_{iG_j}$. The $f^i_{kL_j}$'s and $f^i_{kG_j}$'s then have to satisfy:

$$\sum_i f^i_{kL_j} = \sum_i f^i_{kG_j} = 1 \tag{3.2}$$

and the distribution of the individuals on the different types in the new cohort, p^N_i is given by:[5]

$$p^N_i = \sum_k p_k \left(r \sum_j p_j f^i_{kL_j} + (1-r) \sum_j \pi_j f^i_{kG_j} \right) \tag{3.3}$$

The change in the frequency of p_i-types over time (between cohorts), $\dot{p}_i$, is then:

$$\dot{p}_i = \sum_k p_k \left(r \sum_j p_j f^i_{kL_j} + (1-r) \sum_j \pi_j f^i_{kG_j} \right) - p_i \tag{3.4}$$

For the π_i's, of course, *mutatis mutandis*, the same expressions obtain. Equation system 3.4 and the corresponding system for the π_i's describe the evolution of the society modeled.

3.2.1 The fitness functions and basic assumptions

We specify the fitness determinants $f^i_{kL_j}$ and $f^i_{kG_j}$ and make some assumptions. We first assume that the type at birth is retained if at the interaction both individuals behave in the same manner. In other words, each individual expects the others to behave in the same manner towards it as it towards them; if this is the case, it is not "surprised" (there is no "dissonance" between expected and actual behavior) and will not see any reason to change its own inherited behavior:[6]

Assumption 3.3 $f^i_{iL_i} = 1,\ \forall i$; $f^1_{1L_2} = f^2_{2L_1} = 1$; $f^1_{1G_1} = 1$; $f^2_{2G_2} = f^2_{2G_3} = 1$; $f^3_{3G_2} = f^3_{3G_3} = 1$

This leaves us with the following encounters to be discussed: (L_1, L_3), (L_1, G_2), (L_1, G_3), (L_2, L_3), (L_2, G_1), (L_3, L_1), (L_3, L_2) and (L_3, G_1). In each of these cases, the individual encountered in the interaction have a different behavior from that of the individual discussed, who hence will be "surprised" and because of that might consider changing its own behavior. Since an individual of the L-group cannot distinguish between individuals of type G_2 and type G_3, we are justified in making the assumption:

[5] Using expectations, we here appeal to the law of large numbers, assuming the size of the different groups to be sufficiently large.

[6] This assumption is, strictly speaking, not necessary. A sufficiently small leakage from these encounters would alter the "trusting" G-equilibrium (see below) into a fully mixed one, moving it towards the right in figure 4.1. The substance of the discussion would not be affected, however. The formal machinery would be made considerably more onerous, though.

ASSUMPTION 3.4 $f^i_{1G_2} = f^i_{1G_3},\ \forall i$

By the same token, an individual of *a priori* type L_3 has no way of knowing if it encounters an individual of type L_1 or L_2, leading to the assumption:

ASSUMPTION 3.5 $f^i_{3L_1} = f^i_{3L_2},\ \forall i$

This leaves us with six distinct expressions for the f's, that we have to specify. We assume that if a behavioral change takes place, it will be to a type that implies the smallest change in behavior leading to a difference in the actual interaction. Specifically, an *a priori* type 1 individual, being cheated by someone from the other group, has a tendency to change its behavior and become a type 2, but not a type 3, but being cheated by someone from the own group, the change would be to type 3 and not to type 2. Similarly, an *a priori* type 2 individual, being cheated by someone from the own group, might alter the behavior to pattern 3 but not to pattern 1 and being shown trust from someone from the other group, might change to pattern 1 but not to pattern 3. Also, an *a priori* type 3 person, being shown trust from someone from the own group, changes to pattern 2 with a certain probability, but not to type 1; only in the case of being shown trust from someone from the other group it changes to pattern 1 with some probability but not to pattern 2. That is, we assume:

ASSUMPTION 3.6 $f^2_{1L_3} = f^3_{1G_2} = f^1_{2L_3} = f^3_{2G_1} = f^1_{3L_1} = f^2_{3G_1} = 0$

Finally, a type 1 or type 2 individual encountering a type 3 individual from the same group will suffer the same shock and behave in the same manner. The same holds for a type 2 or type 3 individual encountering a type 1 individual from the other group. That is:

ASSUMPTION 3.7 $f^3_{1L_3} = f^3_{2L_3}$ and $f^1_{2G_1} = f^1_{3G_1}$

The following four functions, giving the probability of a behavioral change, then describe the dynamics of our model and are written as the product of two functions, one, n, capturing the social norm and one, c_b or c_w, capturing the "behavior dissonance", of retaining the *a priori* behavior:

$$\begin{aligned} f^3_{1L_3}(p,u) &= n(p_3)\, c_w(u_{tt} - u_{tc}) \\ f^2_{1G_2}(p,u) &= n(p_2 + p_3)\, c_w(u_{tt} - u_{tc}) \\ f^1_{2G_1}(p,u) &= n(p_1)\, c_b(u_{cc} - u_{ct}) \\ f^2_{3L_1}(p,u) &= n(p_1 + p_2)\, c_b(u_{cc} - u_{ct}) \end{aligned} \tag{3.5}$$

Assumption 3.6, of course, implies:

$$f^1_{1L_3}(p,u) = 1 - f^3_{1L_3}(p,u) \text{ etc.} \tag{3.6}$$

The function $n(p)$ is monotonically non-decreasing in the argument and describes the influence – or strength – of the social norm adhered to by a fraction p of the population, where the various possible (not necessarily mutually exclusive) norms and their popular strengths are given as:

norm I: "trust everyone" with popular strength p_1

norm II: "trust your own" with popular strength $p_1 + p_2$

norm III: "don't trust the others" with popular strength $p_2 + p_3$

norm IV: "don't trust anyone" with popular strength p_3

The function $c_w(\Delta u_w)$ reflects the perceived cost of being trusting when the partner in the encounter cheats you, the costs of the behavioral dissonance. The material loss due to the "unexpected" behavior of the partner is $\Delta u_w := u_{tt} - u_{tc}$. This loss enforces the desire to imitate the behavior of the partner, becoming a cheating instead of a trusting type. The perceived subjective costs of not altering the behavior are assumed to be increasing in Δu_w, and the probability of changing as a consequence increases with the size of Δu_w.

On the other hand, $c_b(\Delta u_b)$, $\Delta u_b := u_{cc} - u_{ct}$, is the perceived costs of the behavioral dissonance when one expects to be cheated by the partner and is trusted. The unexpected material gain from the encounter is $-\Delta u_b := u_{ct} - u_{cc}$. This gain reduces the strength of the desire to imitate the behavior of the partner, becoming a trusting type instead of a cheating one. The greater is this gain, the less likely is then the change from a cheating to a trusting type. That is, the smaller is this gain (the higher is the – negative – utility difference Δu_b) the more likely is the transformation from the cheating to trusting type. This is captured in c_b.

That is, the arguments of c_w and c_b, the perceived "shock", is the difference between the pay-off of the encounter, had the partner behaved as expected, and the pay-off that was realized do to the "surprising" behavior of the partner.[7] We assume that *with the given values* of the u_{ii}'s, the probability that due to the behavioral dissonance an individual changes in the direction of cheating is higher than the probability that it changes in the direction of showing trust *ceteris paribus* (that is, we assume that it is "easier to turn bad than to turn good"):[8]

Assumption 3.8 $\quad c_w(u_{tt} - u_{tc}) \geq c_b(u_{cc} - u_{ct}) > 0$

We also make two basically technical assumptions. In order to facilitate the analysis, differentiability – and as a consequence, continuity – of the relevant functions allows us to use phase diagrams for the analysis. In addition, it is needed that the social norm be not too sensitive to changes in the p's. We assume it to be inelastic in its argument:

Assumption 3.9 The function $n(p)$ is differentiable.

Assumption 3.9, of corse, implies that the functions in expression 3.5 are continuous and differentiable in the p's.

[7] The two functions measure the size of the "shock" due to the "surprise" behavior, and in that way influence the probability of the individual mutating to another state. An alternative interpretation would be to look at the opportunity costs of the *a priori* actual behavior in comparison to the best response to the two different behavior patterns of the partner, $u_{cc} - u_{tc}$ and 0, as well as the opportunity costs of adopting the behavior of the partner in comparison to the best response, 0 and $u_{ct} - u_{tt}$. Intuitively, this contradicts the limited-rationality spirit of the model, which takes its departure in a *status quo* and models behavioral change as the result of external *stimuli*. The formal analysis would in both interpretations remain the same, however.

[8] This assumption is, of course, implicitly a combination of two different assumptions, one comparing $c_w(x)$ and $c_b(x)$ for all x, for instance assuming $c_w(x) \geq c_b(x) \forall x$, and one other comparing Δu_w and Δu_b, for instance assuming $\Delta u_w \geq \Delta u_b$. The chosen formulation is the minimum needed, in order to analyze the non-trivial cases of our model.

ASSUMPTION 3.10 $\frac{\partial n(p)}{\partial p}\frac{p}{n} < 1$

We finally assume that the influence of the social norm in a homogeneous society, however, is strong enough to be felt. Specifically, we let the social pressure in a homogeneous society outweigh the individual gains from egoistic behavior:

ASSUMPTION 3.11 $0 < n\,(0)\,c_w < n\,(1)\,c_b$

Assumptions 3.8 and 3.11 are the crucial assumptions that make the model interesting and lead to the existence of two distinct equilibria. With assumption 3.8 and without assumption 3.11 no stable "trusting" equilibrium would be possible. Similarly, with assumption 3.11 not having assumption 3.8 we could not be certain that a "cheating" equilibrium exists. This becomes evident in the further analysis.

3.2.2 The equations of motion

Equations 3.4 can now be written as:

$$\begin{aligned}
\dot{p}_1 &= -p_1\,[p_3 r n\,(p_3) + (\pi_2 + \pi_3)\,(1-r)\,n\,(p_2+p_3)]\,c_w \\
&\quad + (p_2+p_3)\,\pi_1\,(1-r)\,n\,(p_1)\,c_b \\
\dot{p}_2 &= p_1\,(\pi_2+\pi_3)\,(1-r)\,n\,(p_2+p_3)\,c_w - p_2\pi_1\,(1-r)\,n\,(p_1)\,c_b \\
&\quad + p_2 p_3 r\,[n\,(p_1+p_2)\,c_b - n\,(p_3)\,c_w] + p_1 p_3 r n\,(p_1+p_2)\,c_b \\
\dot{p}_3 &= -p_3\,\{(p_1+p_2)\,r\,[n\,(p_1+p_2)\,c_b - n\,(p_3)\,c_w] + \pi_1\,(1-r)\,n\,(p_1)\,c_b\}
\end{aligned} \tag{3.7}$$

We have suppressed the arguments of the c-functions. Simple addition of the three equations verifies that $\sum_i \dot{p}_i \equiv 0$. *Mutatis mutandis* we can find the corresponding expressions for π. Using the fact that $p_1 + p_2 + p_3 = 1$ and correspondingly for π, we can limit the phase space to the product of two two-simplices. We can, hence, for each ethnic group eliminate one of the equations when analyzing the dynamics of the model.

4 EQUILIBRIUM

Possible equilibria of the dynamic system and their stability are to be analyzed. We begin by considering the dynamics of only one of the groups treating the other one as exogenous, first looking at the case of no interaction between the two ethnical groups and then investigating how the contact to the external world, the other group, affects the possible equilibria. Finally, we consider the mutual interaction between the ethnic groups, making the parameters of both groups endogenous.

4.1 EQUILIBRIUM WITHIN ONE GROUP

4.1.1 Autarchy

If there is no interaction between the two groups, *i.e.* $r = 1$, equations 3.7 reduce to

$$\dot{p}_1 + \dot{p}_2 = -\dot{p}_3 = -\,(p_1+p_2)\,p_3 r\,[n\,(p_3)\,c_w - n\,(p_1+p_2)\,c_b]\,. \tag{4.1}$$

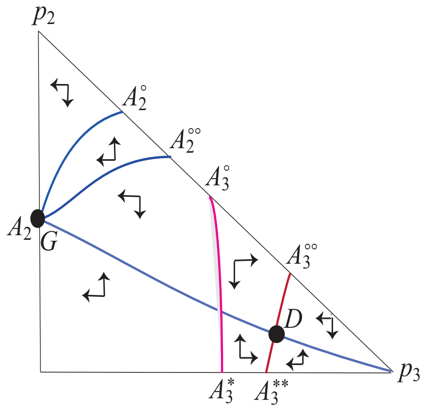

Figure 4.1 Phase diagram for $\dot{p}_2$ and $\dot{p}_3$.

Here, we clearly have a locally stable equilibrium for $p_3 = 1$, since, by assumption 3.8, $c_w \geq c_b$ and since $n(1) \geq n(0)$.[9] A locally stable equilibrium for $p_1 + p_2 = 1$ exists under assumption 3.11. That is, if the influence of the social norm, the social pressure, is sufficiently strong, then a stable equilibrium with only trusting types exists.[10] We formally state:

Proposition 4.1 Under assumptions 3.8 and 3.11, if there is no interaction with the other ethnic group, $r = 1$, two locally stable equilibria exist: the set given by $p_1 + p_2 = 1$ (or $p_3 = 0$), everyone trusts everyone and behaves accordingly, as well as the point given by $p_3 = 1$, everyone cheats everyone.

4.1.2 Interaction

The situation becomes much more involved if $r < 1$; that is, if there is interaction with the other ethnic group. The interaction is captured by the parameters r and π_1 (or $\pi_2 + \pi_3 = 1 - \pi_1$). In appendix A we analyze the dynamics of the p's in this case. An initial result is the following:

Proposition 4.2 For $r < 1$, if $\pi_1 > 0$, there exists no equilibrium $p_3 = 1$.

The dynamics of the system is pictured in a phase diagram in figure 4.1. This demonstrates the existence of at least two locally stable equilibria if π_1 is small enough and r big enough (marked with dots G and D in Figure 4.1). The discussion in appendix A shows that the "right-hand" equilibrium, D, might disappear for a large π_1 or a small r. If π_1 approaches zero or r one, it approaches the point $(0, 0, 1)$. If π_1 approaches one, the "left-side" equilibrium G, which is independent of r, approaches $(1, 0, 0)$ and if π_1 approaches zero, this equilibrium approaches $(0, 1, 0)$.[11] The equilibria are locally stable if π_1 and r are exogenous. We collect this in a proposition:

[9] All equilibria discussed are fixed points of expression 3.3, that is stationary points of equations 3.4, the equations for the dynamics of the system. For the sake of simplicity we will talk about "equilibria" in the text and not in every instant specify that we are dealing with asymptotic fixed points of the dynamic system 3.7.

[10] Due to continuity, there also exists at least one more fixed point of the system. This, however, is an unstable equilibrium. In general, we cannot exclude the existence of further fixed points leading to more equilibria, stable and unstable. In our analysis, however, we will limit the analysis to the case with the minimum number of possible fixed points.

[11] Assuming r to be less than one. If $r = 1$ the G equilibrium is independent of π_1 and equal to the set given by the vertical axis, see proposition 4.1.

PROPOSITION 4.3 If $(0 <) r < 1$, and the behavior of the outside world is exogenous (no feedback), then there exists a stable equilibrium with no individuals of type 3 and a distribution of individuals on type 1 and 2 determined by π_1 (and independent of the value of r), such that the fraction of type one individuals is a monotone function of π_1 with $p_1 = 0$ if $\pi_1 = 0$ and $p_1 = 1$ if $\pi_1 = 1$. Furthermore, for sufficiently small values of π_1 or sufficiently large values of r there exists a second equilibrium with a positive fraction of all three types – as long as π_1 is positive and r is less than one – and with only type 3 individuals if π_1 is zero or r is one.

PROOF Follows from the discussion above and appendix A. ■

We also note that in this case of exogenous influence, as r decreases from 1, that is, contact increases between the two groups, the value of p_3 decreases in the D-equilibrium (the curve $A_3^{**}-A_3^{oo}$ tends to move towards the left) and for some value of r it "jumps" to the G-equilibrium with $p_3 = 0$. Further, the size of the region of the phase diagram for which the G-equilibrium (fixed point) is an attractor (the critical mass for convergence) tends to increase (the curve $A_3^{*}-A_3^{o}$ tends to move towards the right). Somewhat imprecisely, contact tends to make individuals "nicer", or – in other words, to paraphrase Robert Frost – "good fences make suspicious neighbors". Intuitively, as long as there are some trustworthy people in the other group, there will be some individuals who trust the people from the other group and as a consequence, by construction, also the people of their own group. That is, if $\pi_1 > 0$, then also $p_1 > 0$.

4.2 INTERACTION EQUILIBRIA

When the interaction between the two groups is taken into account, we first have to investigate which equilibria are possible and then study the stability properties of those possible equilibria.

4.2.1 Feasible equilibria

We saw in section 4.1.2 that the location of equilibrium G is fully determined by π_1 and equilibrium D by π_1 and r. The coordinates of equilibrium G can hence be written as $(g(\pi_1), 1-g(\pi_1), 0)$ and of equilibrium D as $(d_1(\pi_1,r), d_2(\pi_1,r), d_3(\pi_1,r))$. In the other group we have the corresponding equilibria Γ and Δ with coordinates $(\gamma(p_1), 1-\gamma(p_1), 0)$ and $(\delta_1(p_1,\rho), \delta_2(p_1,\rho), \delta_3(p_1,\rho))$. The following constellations are possible:

- $G-\Gamma$, which requires $p_1 = g(\gamma(p_1))$
- $G-\Delta$, which requires $p_1 = g(\delta_1(p_1))$
- $D-\Gamma$, which requires $p_1 = d_1(\gamma(p_1))$
- $D-\Delta$, which requires $p_1 = d_1(\delta_1(p_1))$

Here, we have suppressed the dependency on the exogenous variables r and ρ. Since $D-\Gamma$ is the mirror image of $G-\Delta$, we have only three fundamentally different constellations, and we have to look for the fixed points of the mappings $g \circ \gamma$, $g \circ \delta_1$, and $d_1 \circ \delta_1$. In appendix B we investigate the fixed points of these mappings and show the existence of at least six feasible equilibria $(p_1, p_2, p_3; \pi_1, \pi_2, \pi_3)$:

(i) $(1,0,0;1,0,0)$

(ii) $(\hat{p},1-\hat{p},0;\hat{\pi},1-\hat{\pi},0)$

(iii) $(0,1,0;0,1,0)$

(iv) $(0,1,0;0,0,1)$

(v) $(0,0,1;0,1,0)$

(vi) $(0,0,1;0,0,1)$

Since (v) is the mirror image of (iv), we do not need to analyze (v) separately.

4.2.2 Stability of the equilibria

In order to find the stability properties of the six equilibria identified above, we rewrite the dynamic system in equations 3.7 as a four-dimensional system, making use of the fact that $p_1 + p_2 + p_3 = 1$ and correspondingly for the π's; that is, we limit the phase space to the product of two two-simplices. We can, hence, in each simplex eliminate the last equation and simplify the first two:

$$\begin{aligned}
\dot{p}_1 &= -p_1\left[\frac{r}{1-r}\left(1-p_1-p_2\right)b\left(1-p_1-p_2\right)+\left(1-\pi_1\right)b\left(1-p_1\right)\right] \\
&\quad + \left(1-p_1\right)\pi_1 a\left(p_1\right) \\
\dot{p}_2 &= p_1\left(1-\pi_1\right)b\left(1-p_1\right)-p_2\pi_1 a\left(p_1\right) \\
&\quad + \left(1-p_1-p_2\right)\left[p_2 e\left(p_1+p_2,1-p_1-p_2\right)+p_1\frac{r}{1-r}a\left(p_1+p_2\right)\right] \\
\dot{\pi}_1 &= -\pi_1\left[\frac{\rho}{1-\rho}\left(1-\pi_1-\pi_2\right)\beta\left(1-\pi_1-\pi_2\right)+\left(1-p_1\right)\beta\left(1-\pi_1\right)\right] \\
&\quad + \left(1-\pi_1\right)p_1\alpha\left(\pi_1\right) \\
\dot{\pi}_2 &= \pi_1\left(1-p_1\right)\beta\left(1-\pi_1\right)-\pi_2 p_1\alpha\left(\pi_1\right) \\
&\quad + \left(1-\pi_1-\pi_2\right)\left[\pi_2\varepsilon\left(\pi_1+\pi_2,1-\pi_1-\pi_2\right)+\pi_1\frac{\rho}{1-\rho}\alpha\left(\pi_1+\pi_2\right)\right]
\end{aligned} \tag{4.2}$$

Here, we have defined:

$$\begin{aligned}
a(p) &:= (1-r)\,n(p)\,c_b \\
b(p) &:= (1-r)\,n(p)\,c_w \\
e\left(p_i,p_j\right) &:= r\left[n\left(p_i\right)c_b - n\left(p_j\right)c_w\right]
\end{aligned} \tag{4.3}$$

For the π-group, the corresponding functions are defined analogously: $\alpha(\pi)$, $\beta(\pi)$, and $\varepsilon\left(\pi_i,\pi_j\right)$.

In appendix C we prove

PROPOSITION 4.4 Of the six equilibria above, all with the exception of number (ii) are locally stable.

Again, we can also look at the global stability properties of the equilibria in a slightly imprecise manner, assuming the interaction in the short run to be exogenous as in section 4.1.2. If π_1 equals zero, the value of r does not influence the size of the area for which the points $(0, 1, 0)$ and $(0, 0, 1)$ in the L-diagram are attractors;[12] in other words, if the outsiders show no trust, then the degree of contact has no effect on the global stability of the society of the insiders. If π_1 is equal to one, however, the size of the area of the phase diagram for which the point $(1, 0, 0)$ in the L-diagram is an attractor tends to be greater the smaller is r.[13] Loosely speaking, equilibrium (i) is more likely to withstand external shocks the smaller is r. The general conclusion above that “good fences make suspicious neighbors” should be amended by “low fences make good neighbors”.

4.2.3 Discussion

The model of the EHMGs corresponds to equilibria (iii), (iv) and (v). We have thus shown that the stability of the EHMG model can be explained from a social-evolution perspective. However, there are at least two other locally stable equilibria. An interesting question is how the selection between the equilibria is made.

There is a clear difference between equilibrium (i), on one side, and equilibria (iii), (iv), (v) and (vi) on the other side. Equilibrium (i) is only feasible if the rest of the world also behaves according to norm I. Hence, it is very sensitive to structural changes outside of the own community. It is nevertheless remarkable that if norm I has any followers in the outside world, then norm IV can not be the sole domestic norm

The selection between equilibria (v) or (vi), on the one hand, and equilibria (iii) or (iv), on the other hand, is not sensitive to the behavior of the outside world. This is, of course, due to the fact that types 2 and 3 are indistinguishable to the outside world. That is, the selection of behavior 2 or 3 is basically a domestic matter. Both behavioral patterns can be stable independently of the prevailing norm in the rest of the world, as long as it isn’t norm I.

Where a society ends up depends on its point of departure and on exogenous structural changes in society. Such changes can be caused by external (or internal) shocks. This is further discussed in HOLLER and WICKSTRÖM (1999). We have, however, seen that there are indications that a high degree of contact, a small r, can make the “good” equilibrium more likely in the sense that the system will end up there for more initial values of the point of departure of the social evolution. In addition, an sensible internalization of r and ρ, assuming them to decrease with the number of encounters involving trusting types from the other group, that is with π_1 and p_1, respectively, will also make the “trusting” equilibrium (i) more likely.

5 CONCLUDING REMARKS

The level of trust in a society characterized by heterogenity in the form of identifiable ethnic or linguistic groups, of which the informal trading networks EHMG are but one manifestation, is very important especially if formal legal structures are weak or absent. We observe such situations not only in less developed economies, but also in former socialist economies in transition

[12] The curve describing the loci of repellents is in this case implicitly given by the equation $n(1-p_3)c_b = n(p_3)c_w$. This, of course, has a unique solution, say p_3^*, such that for an initial value p_3^0 of p_3, $p_3^0 > p_3^*$ implies that $(0, 0, 1)$ is an attractor and $p_3^0 < p_3^*$ implies that $(0, 1, 0)$ is an attractor, independently of the initial values of p_1 and p_2.

[13] This is shown in a more precise manner in appendix A.

to market economies. Also in market economies with well established legal structures the emergence of new heterogenities as a result of globalizing tendencies resulting in integration of local economies or migration the social interactions are becoming more important for the smooth functioning and cohesion of society.

Thus, it is very important to understand why certain patterns of behavior emerge and persist over time. Our paper contributes to the understanding of the persistence and stability of various behavioral patterns by providing a formal model of cultural transmission, in which behavioral norms are transmitted between generation and then modified by imitation in social interactions.

Especially the recent flow of migrants and refugees all over the world has given questions related to these new minorities a certain urgency. The model suggests that perhaps more frequent contacts is part of the solution leading to a more cohesive and more harmonious multi-ethic society. The model also suggests that a well-developed legal infrastructure, especially for contract enforcement, is very important for creating a more harmonious multi-ethnic society because individuals in various interactions then need to put less weight on trust due to the identity of the partner, and can put more weight on trust generated by the impersonal legal infrastructure.

Of course, the model is very stylized and only captures the big picture. Various extensions, permitting more "noise" in the transmission of behavioral patterns and their alteration through contacts could be the topic of further work.

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APPENDICES

A THE DYNAMICS IN A PHASE DIAGRAM

In this appendix we analyze how r and π_1 influence the equilibrium values of the p's. We find the dynamics in the $p_2 - p_3$-dimensions of the simplex $p_1 + p_2 + p_3 = 1$.

A.1 THE DYNAMICS OF p_3

The stationary points of p_3 are found by solving the equation $\dot{p}_3 = 0$. It is clear from the equations of motion, 3.7, that $p_3 = 0$ defines a set of stationary points for p_3. It is also readily seen that these points are locally attracting, by evaluating $\dot{p}_3/p_3$ at $p_3 = 0$:

$$\frac{\dot{p}_3}{p_3} = -r\left[n(1)\,c_b - n(0)\,c_w\right] - \pi_1(1-r)\,n(1-p_2)\,c_b \tag{A.1}$$

Under assumption 3.11, this is negative and, hence, $p_3 = 0$ is a stable value. If $p_3 = 1$, the equation of motion reduces to:

$$\dot{p}_3 = -\pi_1(1-r)\,n(0)\,c_b \tag{A.2}$$

This is negative, unless $\pi_1 = 0$. The stable equilibrium $p_3 = 1$ in the case of no interaction is thus altered if in the outside world there is a positive probability of meeting an individual that can be trusted ($\pi_1 > 0$), which is stated in proposition 4.2.

If there are other stationary points of p_3 than $p_3 = 0$, then they have to solve equation:

$$\frac{1-p_3}{n(1-p_2-p_3)}\left[n(p_3)\,c_w - n(1-p_3)\,c_b\right] = \pi_1\frac{1-r}{r}c_b \tag{A.3}$$

For a given value of p_2 and a sufficiently small π_1, this equation has at least two roots under assumptions 3.8 and 3.11. For a sufficiently large π_1 and a sufficiently small r, it might have no

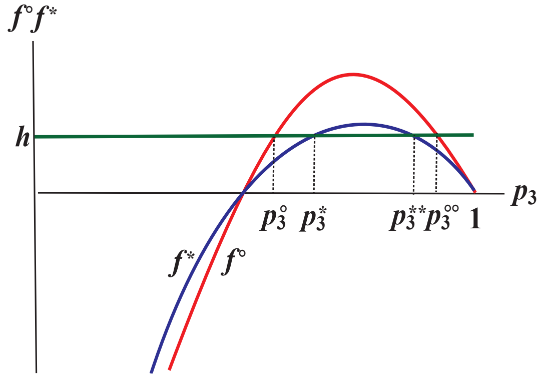

FIGURE A.1 Stationary points of p_3 for some values of π_1 and r, $0 < \pi_1 < 1$ and $0 < r < 1$.

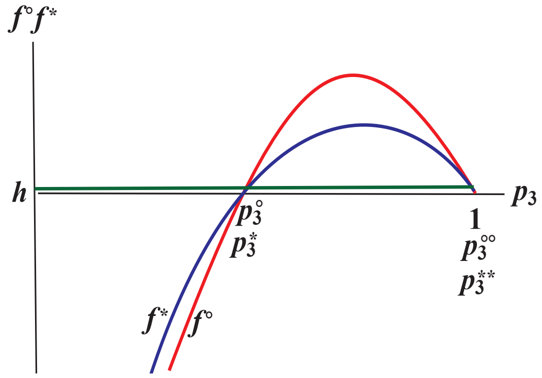

Figure A.2 Stationary points of p_3 as $\pi_1 \to 0$ or $r \to 1$.

root, and $\dot{p}_3 < 0$ everywhere. The case with two roots has been pictured in the phase diagram, figure 4.1. The lines $\left(A_3^*, A_3^o\right)$ and $\left(A_3^{**}, A_3^{oo}\right)$ are the loci of stationary points for p_3. The first one is a repellent and the second one an attractor. The p_3 coordinates of points A_3^* and A_3^{**}, p_3^* and p_3^{**}, solve equation:

$$\frac{1-p_3}{n\left(1-p_3\right)}\left[n\left(p_3\right)c_w - n\left(1-p_3\right)c_b\right] = \pi_1 \frac{1-r}{r} c_b \tag{A.4}$$

The p_2 coordinate is, of course, zero. The p_3 coordinates of A_3^o and A_3^{oo}, p_3^o and p_3^{oo}, solve equation:

$$\frac{1-p_3}{n\left(0\right)}\left[n\left(p_3\right)c_w - n\left(1-p_3\right)c_b\right] = \pi_1 \frac{1-r}{r} c_b \tag{A.5}$$

The p_2 coordinates are $1-p_3^o$ and $1-p_3^{oo}$, respectively.

The behavior of these stationary points can be visualized in simple diagrams. Define $f^*(p_3)$,

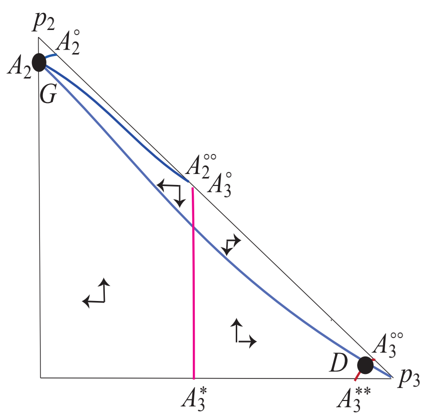

Figure A.3 Phase diagram for $\dot{p}_2$ and $\dot{p}_3$ as $\pi_1 \to 0$ and $0 < r < 1$.

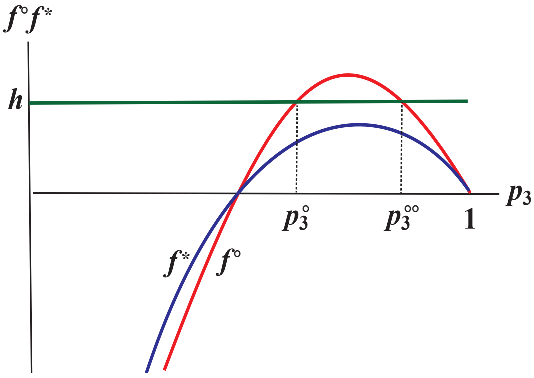

FIGURE A.4 Stationary points of p_3 for sufficiently large values of π_1 or sufficiently small values of r, $0 < \pi_1 < 1$ and $0 < r < 1$.

$f^o(p_3)$, and h as:

$$f^*(p_3) := \frac{1-p_3}{n(1-p_3)}\left[n(p_3)c_w - n(1-p_3)c_b\right]$$
$$f^o(p_3) := \frac{1-p_3}{n(0)}\left[n(p_3)c_w - n(1-p_3)c_b\right] \tag{A.6}$$
$$h := \pi_1 \frac{1-r}{r}c_b$$

We note that:

$$f^o(p_3) = \frac{n(1-p_3)}{n(0)}f^*(p_3) \tag{A.7}$$

That is, $|f^o(p_3)| \geq |f^*(p_3)|$ and they are equal if $p_3 = 1$ or if the expression in the square brackets is zero. In figures A.1, A.2, and A.4 the stationary points of p_3 are indicated for different values of h. Figure A.1 corresponds to the situation in the phase diagram in figure 4.1 with three distinct regions of relevance for the dynamics of p_3, one with an increasing and two with a decreasing p_3.

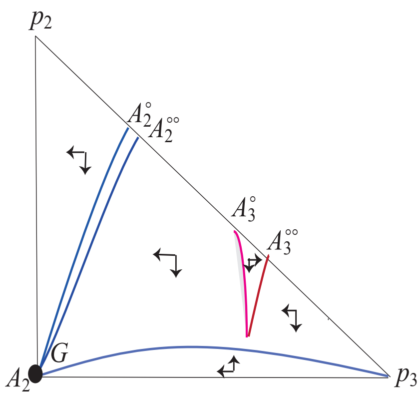

FIGURE A.5 Phase diagram for $\dot{p}_2$ and $\dot{p}_3$ as $\pi_1 \to 1$ and r is sufficiently small.

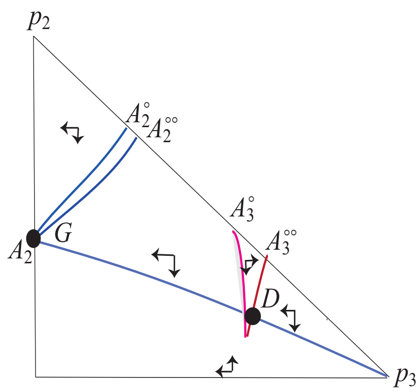

FIGURE A.6 Phase diagram for $\dot{p}_2$ and $\dot{p}_3$ for $0 < \pi_1 < 1$ and r sufficiently small.

In figure A.2 the situation is pictured when π_1 approaches zero or r approaches one. Now, p_3^{**} and p_3^{oo} approach one and p_3^{*} and p_3^{o} become identical, in fact, the curve (A_3^{*}, A_3^{o}) becomes a straight line and the curve (A_3^{**}, A_3^{oo}) degenerates to the point $p_3 = 1$. The equilibrium in dominant strategies in a world without interaction is a limiting case of an equilibrium in a world with vanishing interaction. The resulting phase diagram with two regions for the dynamics of p_3 is pictured in figure A.3.

In figure A.4 p_3^{*} and p_3^{**} disappear and eqilibrium D disappears, too, figure A.5, or becomes globally very unstable, figure A.6, depending on the size of π_1.

A.2 THE DYNAMICS OF p_2

The equation of motion for p_2 is:

$$\begin{aligned}\dot{p}_2 &= (1-p_2-p_3)(1-\pi_1)(1-r)\,n(p_2+p_3)\,c_w \\ &\quad - p_2\pi_1(1-r)\,n(1-p_2-p_3)\,c_b \\ &\quad + p_2p_3r\left[n(1-p_3)\,c_b - n(p_3)\,c_w\right] + (1-p_2-p_3)\,p_3rn(1-p_3)\,c_b\end{aligned} \tag{A.8}$$

If $p_2 = 0$, it reduces to:

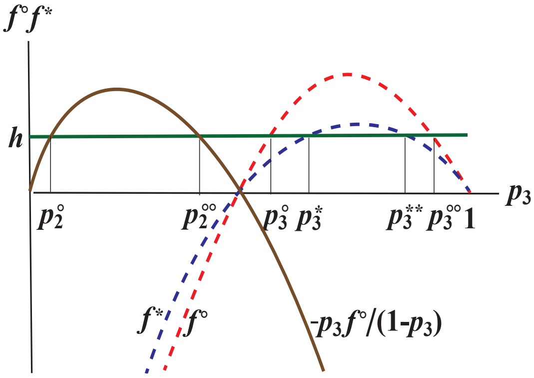

FIGURE A.7 Stationary points of p_2 (and p_3) for some values of π_1 and r, $0 < \pi_1 < 1$ and $0 < r < 1$.

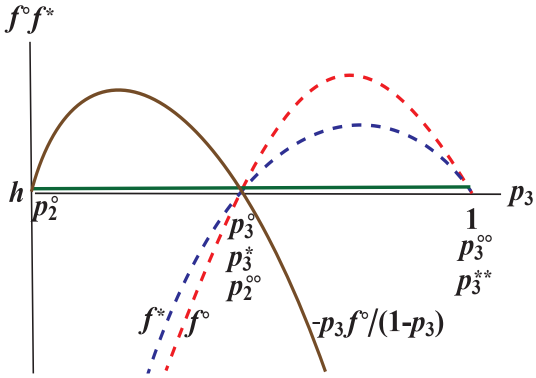

FIGURE A.8 Stationary points of p_2 (and p_3) as $\pi_1 \to 0$ or $r \to 1$.

$$\dot{p}_2 = (1 - p_3)\left[(1 - \pi_1)(1 - r)\, n(p_3)\, c_w + p_3 r n(1 - p_3)\, c_b\right] \tag{A.9}$$

This is positive if $p_3 < 1$. For $p_2 = 1$ we similarly find:

$$\dot{p}_2 = -\pi_1 (1 - r)\, n(0)\, c_b \tag{A.10}$$

This is negative, unless $\pi_1 = 0$. Hence, due to continuity, there is a region in the phase diagram for sufficiently small values of p_2, where p_2 is growing and a region for sufficiently high values of p_2, where p_2 is getting smaller. Along the edges of the phase diagram, for $p_3 = 0$ and $p_2 + p_3 = 1$, we can find expressions for the limits of these regions. If $p_3 = 0$, we find:

$$\dot{p}_2 = (1 - p_2)(1 - \pi_1)(1 - r)\, n(p_2)\, c_w - p_2 \pi_1 (1 - r)\, n(1 - p_2)\, c_b \tag{A.11}$$

and for $p_2 + p_3 = 1$:

$$\dot{p}_2 = -p_2 \pi_1 (1 - r)\, n(0)\, c_b + p_2 p_3 r\left[n(1 - p_3)\, c_b - n(p_3)\, c_w\right] \tag{A.12}$$

The stationary points on the $p_3 = 0$ axis are given by the solution of:

$$\frac{1 - p_2}{n(1 - p_2)} \frac{n(p_2)}{p_2} = \frac{\pi_1}{1 - \pi_1} \frac{c_b}{c_w} \tag{A.13}$$

Under assumption 3.10, this equation has a unique solution, p_2^*, which does not depend on r and which is monotone in π_1 and equal to one if π_1 is zero and zero if π_1 is one. The stationary points for $p_2 + p_3 = 1$ are at $p_2 = 0$ or given as the solutions to the equation:

$$\pi_1 \frac{1 - r}{r} c_b = \frac{p_3}{n(0)}\left[n(1 - p_3)\, c_b - n(p_3)\, c_w\right] \tag{A.14}$$

This can be rewritten as:

$$h = -\frac{p_3}{1 - p_3} f^o(p_3) \tag{A.15}$$

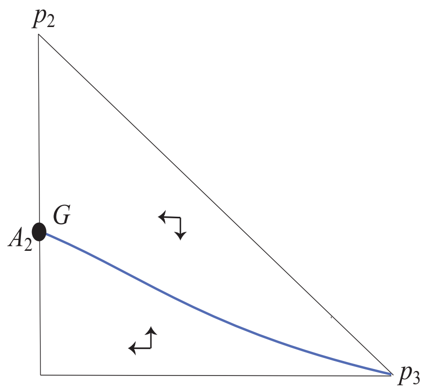

FIGURE A.9 Phase diagram for $\dot{p}_2$ and $\dot{p}_3$ for sufficiently small r.

If π_1 is sufficiently small and r sufficiently big, this equation will have at least two roots, which are the p_3 coordinates of the points A_2^g and A_2^{go}. It is readily seen that these points will lie to the left of the points A_3^g and A_3^{go} in the phase diagram in figure 4.1, since the expression in the square brackets will have to have the opposite sign in the expressions for the stationarity of p_2 and p_3.

In figures A.7 and A.8 we have indicated, how the stationary points can be found. Figure A. 7 corresponds to phase diagram 4.1. Similarly, in figure A.8 the limiting situation with $h \to 0$, corresponding to phase diagram A.3 is illustrated.

A.3 GLOBAL STABILITY

An inspection of the various phase diagrams clearly indicates the smaller is r the more attractive globally becomes equilibrium G. For sufficiently small values of r the D equilibrium even disappears. Furthermore, for sufficiently large values of h (small r and/or large π_1), the phase diagram consists of only two regions and G is the only equilibrium, the location of which is determined by the value of π_1, see phase diagram A.9. Hence, contact feeds trust.

B FEASIBLE INTERACTION EQUILIBRIA

We will show the existence of fixed points of the mappings in section 4.2.1. We show several lemmata.

LEMMA B.1 The mapping $g(\gamma(p))$ has at least three fixed points, $p = 0$, $p = 1$, and $p = \hat{p} \in (0, 1)$.

PROOF First two are trivial. For the third one, we note that the function g is monotone and implicitly defined by

$$\frac{g(\pi_1)}{1 - g(\pi_1)} = \frac{\pi_1}{1 - \pi_1} \frac{n\,(g(\pi_1))\,c_b}{n\,(1 - g(\pi_1))\,c_w} \tag{B.1}$$

and γ by

$$\frac{\gamma(p_1)}{1 - \gamma(p_1)} = \frac{p_1}{1 - p_1} \frac{\nu\,(\gamma(p_1))\,\kappa_b}{\nu\,(1 - \gamma(p_1))\,\kappa_w}. \tag{B.2}$$

The function g has a fixed point $\check{p}$ given by $n(\check{p})c_b = n(1-\check{p})\,c_w$. In view of assumption 3.11, this point exists, and it is unique if $n(p)$ is strictly monotone. By the same argument we find the fixed point $\check{\pi}$ of γ. If $1 > p > \check{p}$, then $g(p) < p$, and if $0 < p < \check{p}$, then $g(p) > p$. The corresponding result holds for γ and $\check{\pi}$. Combining these results, we find that if $1 > p > \max\{\check{p}, \check{\pi}\}$, then $g(\gamma(p)) < p$, and if $0 < p < \min\{\check{p}, \check{\pi}\}$, then $g(\gamma(p)) > p$. By continuity, the fixed point $\hat{p} = g\,(\hat{\pi})$ exists and $\min\{\check{p}, \check{\pi}\} \leq \hat{p} = g\,(\hat{\pi}) \leq \max\{\check{p}, \check{\pi}\}$, and by the same token $\hat{\pi} = \gamma(\hat{p})$ also satisfies $\{\check{p}, \check{\pi}\} \leq \hat{\pi} = \gamma(\hat{p}) \leq \max\{\check{p}, \check{\pi}\}$. ■

REMARK B.1 If the two groups are identical, $\hat{p}$ is unique and equal to $\check{p} = \check{\pi}$.

LEMMA B.2 The mapping $g(\delta_1(p))$ has at least one fixed point, $p = 0$.

PROOF Trivial. ■

LEMMA B.3 The mapping $d_1(\delta_1(p))$ has at least one fixed point, $p = 0$.

PROOF Trivial. ■

There exist then at least the six feasible equilibria listed in section 4.2.1.

C PROOF OF PROPOSITION 4.2.2

We linearize the system 4.2 at the various equilibria and calculate the eigenvalues of the determinants of the matrices of partial derivatives of the equations of motion. The matrices of partial derivatives are given in appendix D. The eigenvalues associated with the various equilibria are:

Equilibrium (i):

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} -a - \frac{r}{1-r}(a-b) < 0 \\ -\alpha - \frac{\rho}{1-\rho}(\alpha-\beta) < 0 \\ \frac{1}{2}\left(-a-\alpha+\sqrt{(a+\alpha)^2 - 4\,(a\alpha-\beta b)}\right) < 0 \\ \frac{1}{2}\left(-a-\alpha-\sqrt{(a+\alpha)^2 - 4\,(a\alpha-\beta b)}\right) < 0 \end{pmatrix}, \tag{C.1}$$

where $a = a\,(1)$, $\alpha = \alpha\,(1)$, $b = b\,(0)$, $\beta = \beta\,(0)$. Since $a\,(1) > b\,(0) > 0$ and $\alpha\,(1) > \beta\,(0) > 0$, all four eigenvalues are real and negative. Hence, equilibrium (1) is locally stable.

Equilibrium (iii):

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} -e\,(1,0) < 0 \\ -\epsilon\,(1,0) < 0 \\ \frac{1}{2}\left(-\beta-b+\sqrt{(\beta+b)^2 - 4\,(\beta b-\alpha a)}\right) < 0 \\ \frac{1}{2}\left(-\beta-b-\sqrt{(\beta+b)^2 - 4\,(\beta b-\alpha a)}\right) < 0 \end{pmatrix}, \tag{C.2}$$

where $a = a(0)$, $\alpha = \alpha(0)$, $b = b(1)$, $\beta = \beta(1)$. Since $b(1) > a(0) > 0$ and $\beta(1) > \alpha(0) > 0$, all four eigenvalues are real and negative. Hence, equilibrium (3) is locally stable.

Equilibrium (iv):

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} -e(1,0) < 0 \\ \varepsilon(0,1) < 0 \\ \frac{1}{2(1-\rho)}\left(-w + \sqrt{w^2 - 4(1-\rho)\left[b\beta - \alpha a(1-\rho)\right]}\right) < 0 \\ \frac{1}{2(1-\rho)}\left(-w - \sqrt{w^2 - 4(1-\rho)\left[b\beta - \alpha a(1-\rho)\right]}\right) < 0 \end{pmatrix}, \tag{C.3}$$

where $a = a(0)$, $\alpha = \alpha(0)$, $b = b(1)$, $\beta = \beta(1)$, and $w := b(1-\rho) + \beta$. Since $b(1) > a(0) > 0$ and $\beta(1) > \alpha(0) > 0$, all four eigenvalues are real and negative. Hence, equilibrium (4) – and, by symmetry, equilibrium (5) – is locally stable.

Equilibrium (vi):

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} e(0,1) < 0 \\ \varepsilon(0,1) < 0 \\ \frac{1}{2\omega}\left(-w + \sqrt{w^2 - 4\omega\left[b\beta - \alpha a\omega\right]}\right) < 0 \\ \frac{1}{2\omega}\left(-w - \sqrt{w^2 - 4\omega\left[b\beta - \alpha a\omega\right]}\right) < 0 \end{pmatrix}, \tag{C.4}$$

where $a = a(0)$, $\alpha = \alpha(0)$, $b = b(1)$, $\beta = \beta(1)$, $\omega = (1-r)(1-\rho)$, and $w := b(1-\rho) + \beta(1-r)$. Since $b(1) > a(0) > 0$ and $\beta(1) > \alpha(0) > 0$, all four eigenvalues are real and negative. Hence, equilibrium (6) is locally stable.

Equilibrium (ii):

Here, in order to make the problem tractable, we have to make the assumption that both groups have identical functions describing their behavior. That is, $a(x) = \alpha(x)$, $b(x) = \beta(x)$, etc., which, of course, implies that $\hat{p} = \hat{\pi}$. Then the eigenvalues are:

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} = \begin{pmatrix} d > 0 \\ -\hat{p}a(\hat{p}) - \frac{r}{1-r}\left[a(1) - b(0)\right] < 0 \\ -\hat{p}a(\hat{p}) - \frac{r}{1-r}\left[a(1) - b(0)\right] < 0 \\ -2a(\hat{p}) + d < 0 \end{pmatrix}, \tag{C.5}$$

where $d := \hat{p}(1-\hat{p})(1-r)\left[\frac{\partial n(1-\hat{p})}{\partial(1-p)}c_w + \frac{\partial n(\hat{p})}{\partial p}c_b\right]$. The last inequality follows from the fact that, by assumption 3.10, $a(\hat{p}) > d$.

The positive eigenvalue d is associated with the eigenvector

$$\begin{pmatrix} e_{p_1} \\ e_{p_2} \\ e_{\pi_1} \\ e_{\pi_2} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}. \tag{C.6}$$

Hence, a perturbation increasing (or decreasing) p_1 and π_1 by a small amount, holding $p_1 + p_2$ and $\pi_1 + \pi_2$ constant (which is a movement in the two simplices) will cause a movement away from the equilibrium (2). Equilibrium (2) is thus unstable.

D THE MATRICES OF PARTIAL DERIVATIVES OF THE EQUATIONS OF MOTION

In this appendix, we give the matrices of the partial derivatives of the equations of motion, equations 4.2. For the definitions of the various parameters, we refer to the discussion in the main text.

For equilibrium (i), $(1,0,0;1,0,0)$, we have:

$$\begin{pmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial \pi} \\ \frac{\partial \dot{\pi}}{\partial p} & \frac{\partial \dot{\pi}}{\partial \pi} \end{pmatrix} = \begin{pmatrix} \frac{r}{1-r}b - a & \frac{r}{1-r}b & b & 0 \\ -\frac{r}{1-r}a & -\frac{1}{1-r}a & -b & 0 \\ \beta & 0 & \frac{\rho}{1-\rho}\beta - \alpha & \frac{\rho}{1-\rho}\beta \\ -\beta & 0 & -\frac{\rho}{1-\rho}\alpha & -\frac{1}{1-\rho}\alpha \end{pmatrix}, \tag{D.1}$$

for equilibrium (ii), $(\hat{p}, 1-\hat{p}, 0; \hat{\pi}, 1-\hat{\pi}, 0)$:

$$\begin{pmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial \pi} \\ \frac{\partial \dot{\pi}}{\partial p} & \frac{\partial \dot{\pi}}{\partial \pi} \end{pmatrix} = \begin{pmatrix} \Psi & \begin{matrix} a(\hat{p}) & 0 \\ -a(\hat{p}) & 0 \end{matrix} \\ \begin{matrix} a(\hat{p}) & 0 \\ -a(\hat{p}) & 0 \end{matrix} & \Psi \end{pmatrix}, \tag{D.2}$$

where

$$\Psi := \begin{pmatrix} -a(\hat{p}) + d + \hat{p}\frac{r}{1-r}b\,(0) & \hat{p}\frac{r}{1-r}b\,(0) \\ \left[(1-\hat{p})a(\hat{p}) - \frac{r}{1-r}a\,(1) \right. & \left[-\hat{p}a(\hat{p}) - \frac{r}{1-r}a\,(1) \right. \\ \left. +(1-\hat{p})\frac{r}{1-r}b\,(0) - d\right] & \left. +(1-\hat{p})b\,(0)\right] \end{pmatrix}, \tag{D.3}$$

for equilibrium (iii), $(0,1,0;0,1,0)$:

$$\begin{pmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial \pi} \\ \frac{\partial \dot{\pi}}{\partial p} & \frac{\partial \dot{\pi}}{\partial \pi} \end{pmatrix} = \begin{pmatrix} -b & 0 & a & 0 \\ b-e & -e & -a & 0 \\ \alpha & 0 & -\beta & 0 \\ -\alpha & 0 & \beta - \epsilon & -\epsilon \end{pmatrix}, \tag{D.4}$$

for equilibrium (iv), $(0,1,0;0,0,1)$:

$$\begin{pmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial \pi} \\ \frac{\partial \dot{\pi}}{\partial p} & \frac{\partial \dot{\pi}}{\partial \pi} \end{pmatrix} = \begin{pmatrix} -b & 0 & a & 0 \\ b-e & -e & -a & 0 \\ \alpha & 0 & -\frac{\beta}{1-\rho} & 0 \\ 0 & 0 & \beta + \frac{\beta}{1-\rho}\alpha & -\delta \end{pmatrix}, \tag{D.5}$$

and, finally, for equilibrium (vi), $(0,0,1;0,0,1)$:

$$\begin{pmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial \pi} \\ \frac{\partial \dot{\pi}}{\partial p} & \frac{\partial \dot{\pi}}{\partial \pi} \end{pmatrix} = \begin{pmatrix} -\frac{b}{1-r} & 0 & a & 0 \\ b + \frac{r}{1-r}a & -d & 0 & 0 \\ \alpha & 0 & -\frac{\beta}{1-\rho} & 0 \\ 0 & 0 & \beta + \frac{\beta}{1-\rho}\alpha & -\delta \end{pmatrix}. \tag{D.6}$$